

# The Full-Sphere Sound Field of Constant Beamwidth Transducer (CBT) Loudspeaker Line Arrays

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# Outline

- Theory
  - ◆ Overview of Constant Beamwidth Transducer Theory
    - ◆ Originally developed for underwater sound by the military
    - ◆ Shaded circular spherical caps
  - ◆ Review: Application of CBT theory to curved-surface and line arrays
- Overview of 3D Sound Radiation Numeric Simulator
  - ◆ Point source arrays
  - ◆ Beamwidth, directivity, loss, polars, footprints, balloon plots
- Straight-Line Array Sound Radiation
- Curved-Line CBT Array Sound Radiation
- Conclusions

# Overview of Constant Beamwidth Transducer Theory

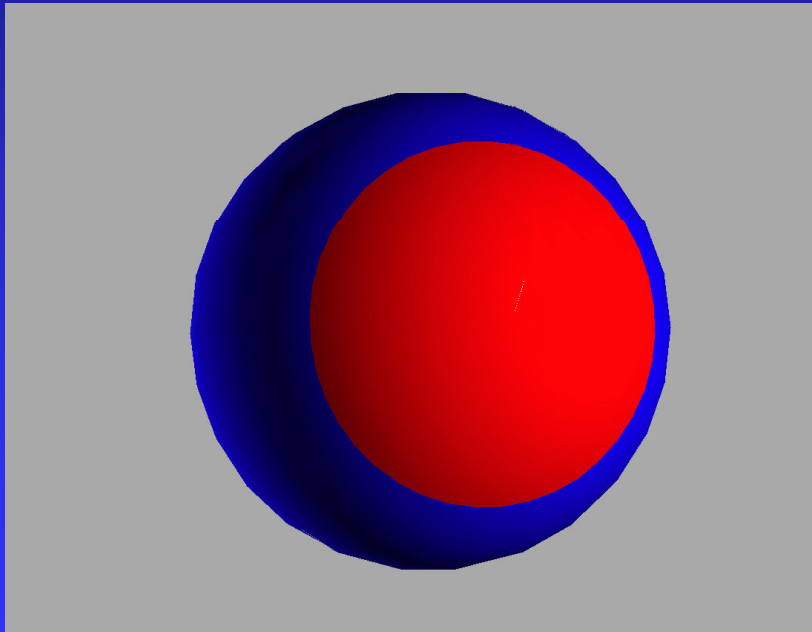
- First formulated in JASA papers published in 1978 and 1983 (U.S. Naval Research Labs) describing underwater transducers based on shaded spherical caps.
  - ◆ P. H. Rogers, and A. L. Van Buren, “New Approach to a Constant Beamwidth Transducer,” J. Acous. Soc. Am., vol. 64, no. 1, pp. 38-43 (1978 July).
  - ◆ A. L. Van Buren, L. D. Luker, M. D. Jevnager, and A. C. Tims, “Experimental Constant Beamwidth Transducer,” J. Acous. Soc. Am., vol. 73, no. 6, pp. 2200-2209 (1983 June).
- Applied to loudspeaker arrays by Keele in 2000 and 2002.
  - ◆ D. B. Keele, Jr., “The Application of Broadband Constant Beamwidth Transducer (CBT) Theory to Loudspeaker Arrays,” presented at the 109<sup>th</sup> convention of the Audio Engineering Society (2000 Sept.) preprint 5216.
  - ◆ D. B. Keele, Jr., “Implementation of Straight-Line and Flat-Panel Constant Beamwidth Transducer (CBT) Loudspeaker Arrays Using Signal Delays,” presented at the 113<sup>th</sup> convention of the Audio Engineering Society (2002 Oct.) preprint 5653.

# Spherical-Cap CBT Transducers

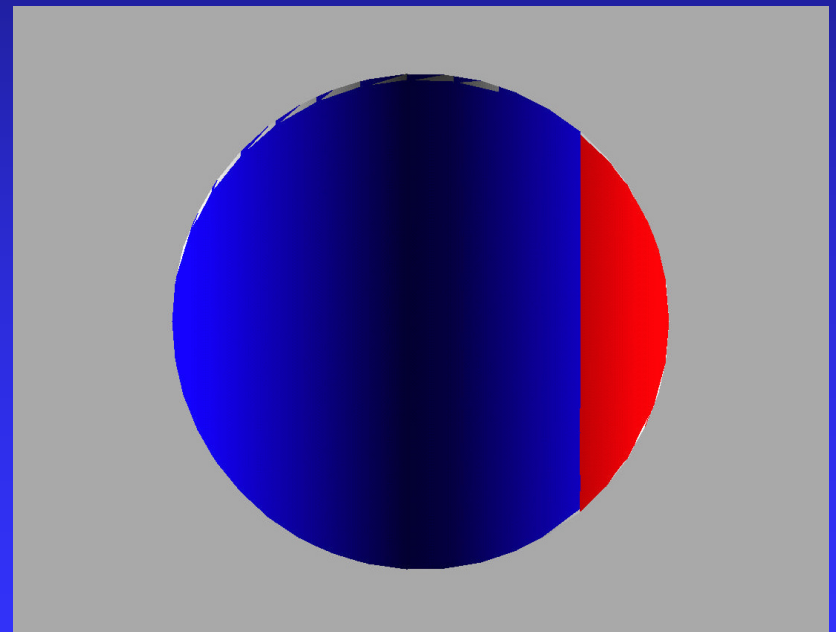
## Overview

100° Circular Spherical Cap

Oblique View



Side View



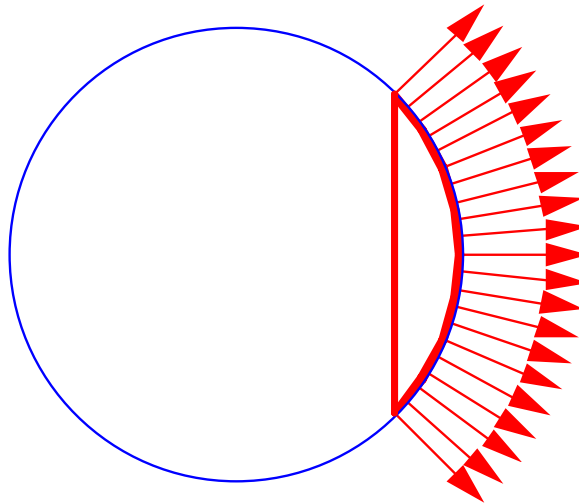
# Spherical-Cap CBT Transducers

## Overview Cont.:

### Legendre Shading of Surface Pressure

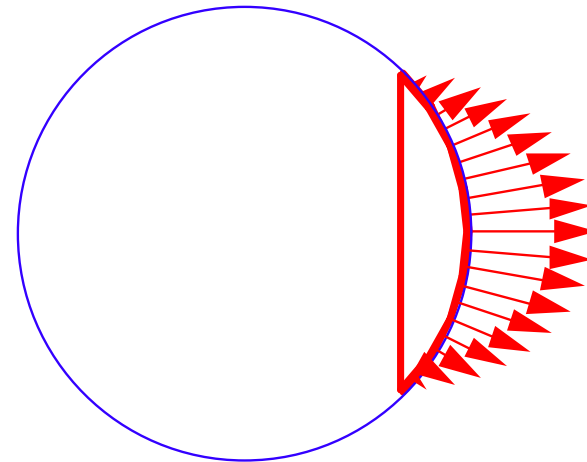
No Shading

$$p(\theta) \propto \begin{cases} 1 & \text{for } \theta \leq \theta_0 \\ 0 & \text{for } \theta > \theta_0 \end{cases}$$



With Shading

$$p(\theta) \propto \begin{cases} P_v(\cos \theta) & \text{for } \theta \leq \theta_0 \\ 0 & \text{for } \theta > \theta_0 \end{cases}$$



# Spherical-Cap CBT Transducers

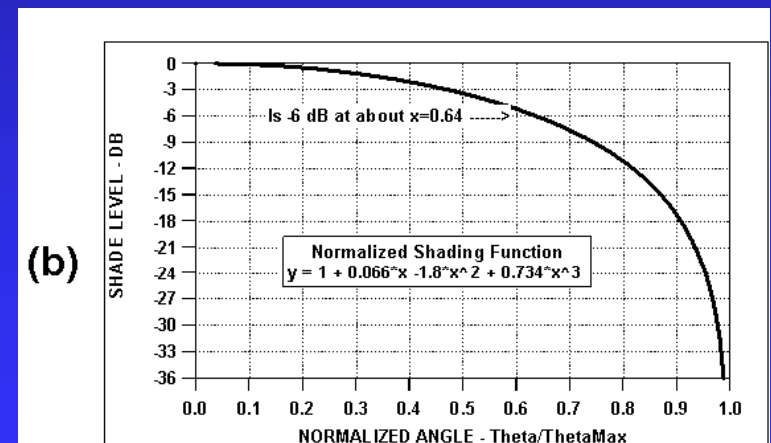
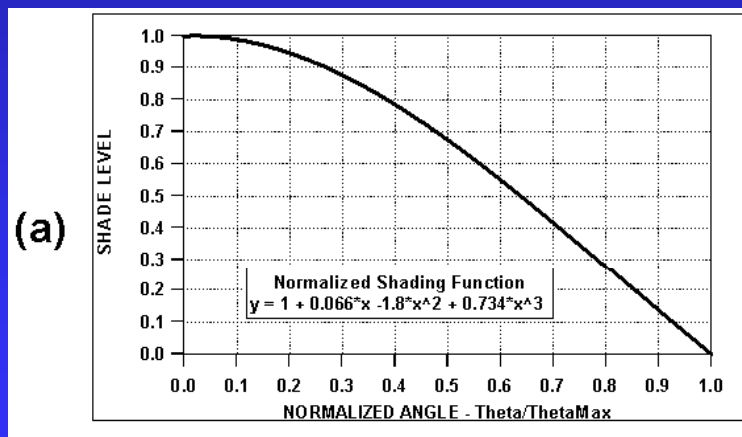
## Overview Cont.: Legendre Shading

- Power Series Approximation of Legendre Shading Function (Works over the whole coverage angle range!)

$$U(x) \approx \begin{cases} 1 + 0.066x - 1.8x^2 + 0.743x^3 & \text{for } x \leq 1 \\ 0 & \text{for } x > 1 \end{cases}$$

where

$$x = \text{normalized angle} \left( \frac{\theta}{\theta_0} \right)$$



# Spherical-Cap CBT Transducers

## Overview Cont.: Observations

- Provides extremely uniform polars above a certain frequency which are independent of distance
- Beamwidth =  $0.64 \times \text{Cap Angle}$
- Surface pressure distribution, nearfield pressure pattern, and farfield pressure pattern are all essentially the same!
- Don't need the rest of the sphere!

# CBT Curved-Surface and Curved-Line Arrays: Review

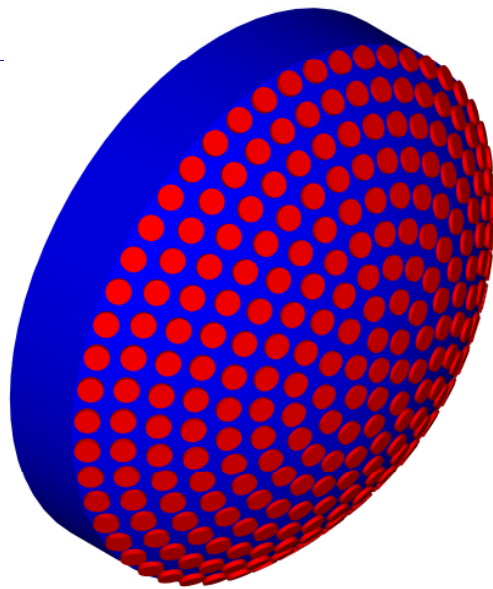
- Can transform a continuous distribution into a discrete distribution
  - ◆ Spherical-cap CBT Arrays
  - ◆ Circular wedge curved-line CBT arrays



# CBT Curved-Surface and Curved-Line Arrays: Review Cont.

## Spherical-cap CBT array example

Oblique View

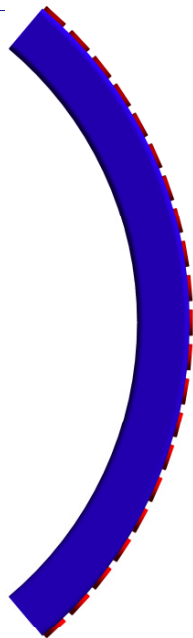


- 266 wide-range drivers
- $100^\circ$  cap angle
- Provides  $64^\circ$  symmetrical vertical and horizontal coverage
- 9 rings

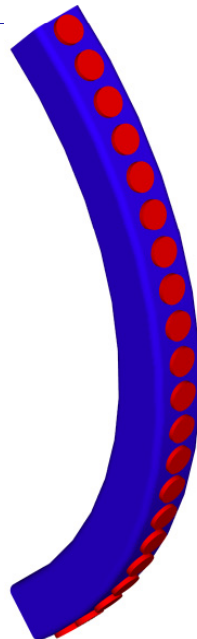
# CBT Curved-Surface and Curved-Line Arrays: Review Cont.

Circular wedge curved-line CBT array example

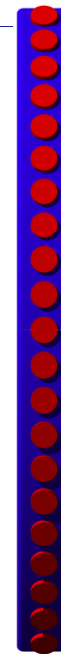
Side View



Oblique View



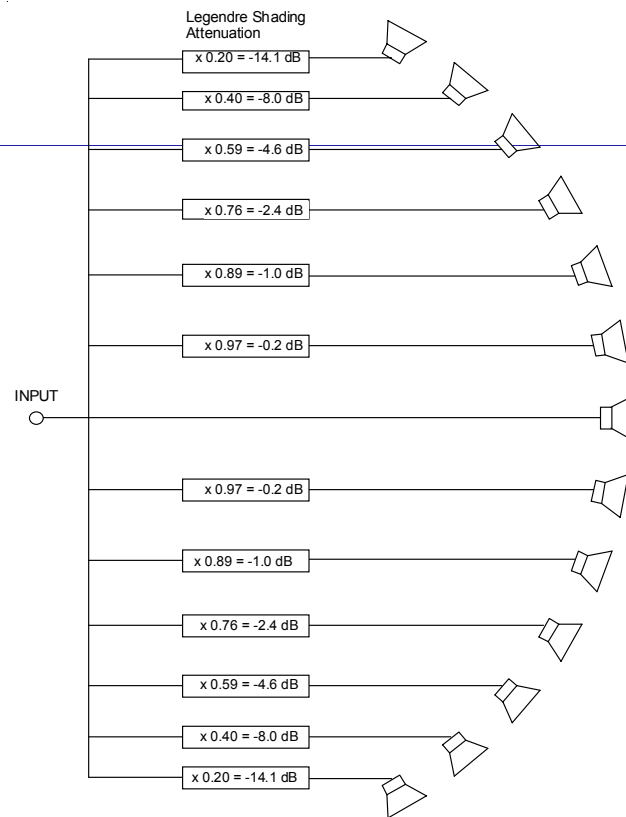
Front View



- 21 wide-range drivers
- 100° wedge angle
- Provides 64° vertical and wide horizontal coverage

# CBT Curved-Surface and Curved-Line Arrays: Review Cont.

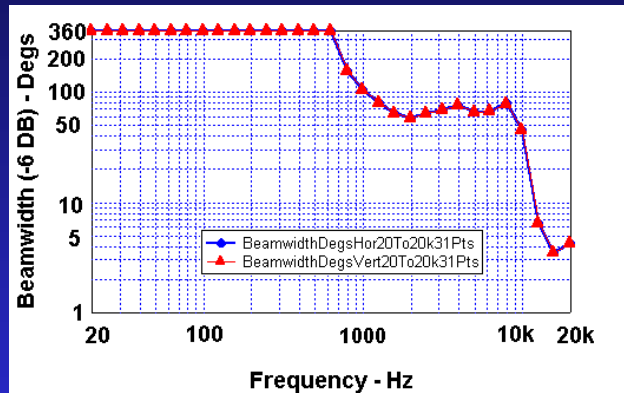
## CBT Signal Processing



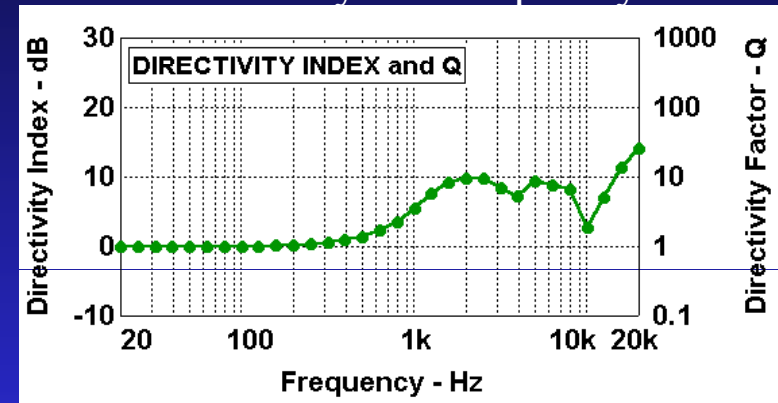
- Each block sets gain only!
- Implements frequency-independent Legendre shading.
- Can be implemented with or without power amplifiers.

# Overview of Point-Source Array 3D Sound Radiation Numeric Simulator

Beamwidth vs. Frequency

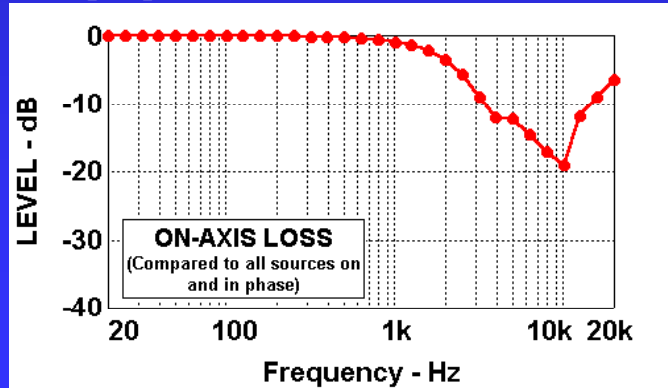


Directivity vs. Frequency

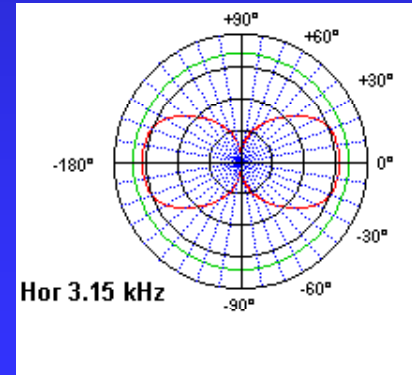


On-Axis Loss vs. Frequency

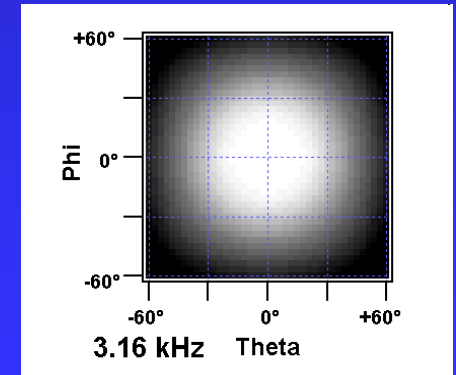
(As compared to all sources on and in phase at sample point.)



Polars  
(Both Vert. And Hor.)



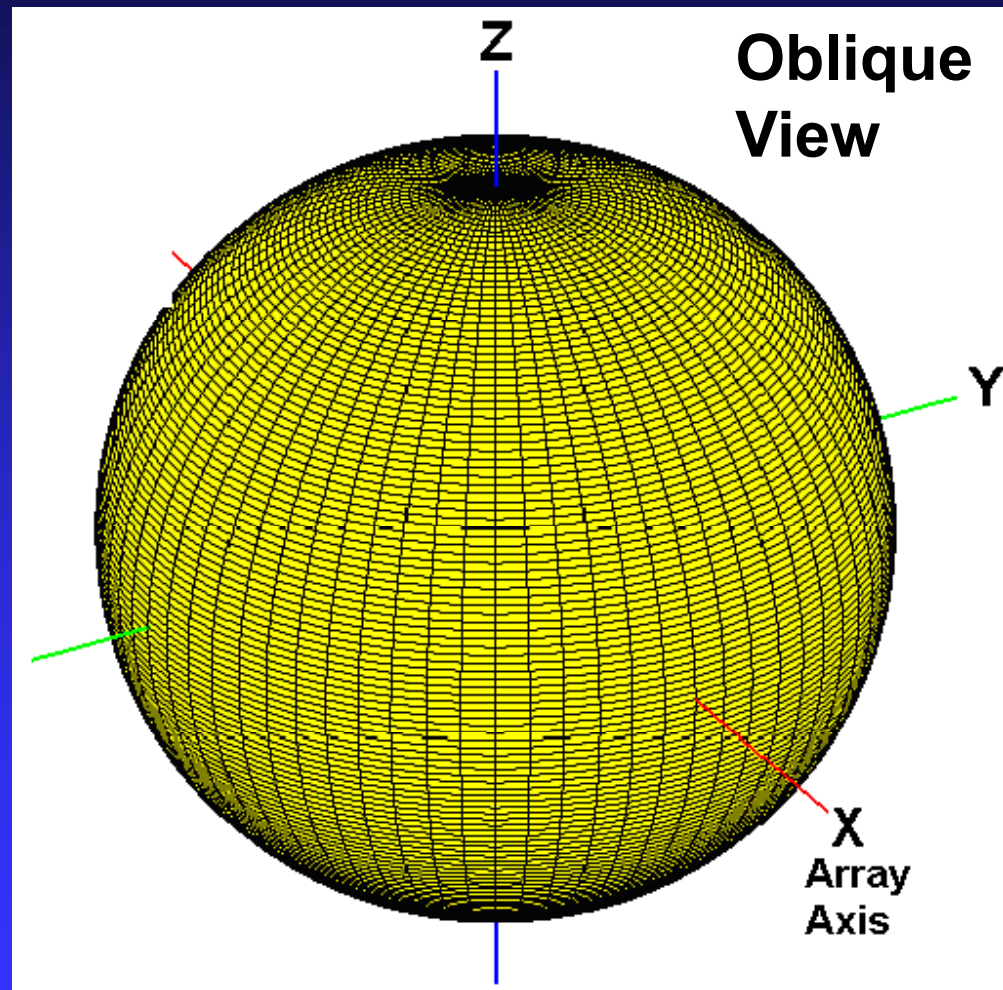
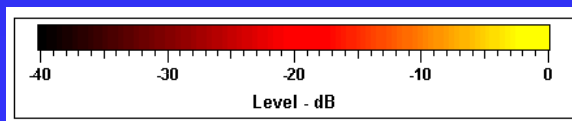
Footprints



# Overview of Point-Source Array 3D Sound Radiation Numeric Simulator: Cont.

## Balloon Plots

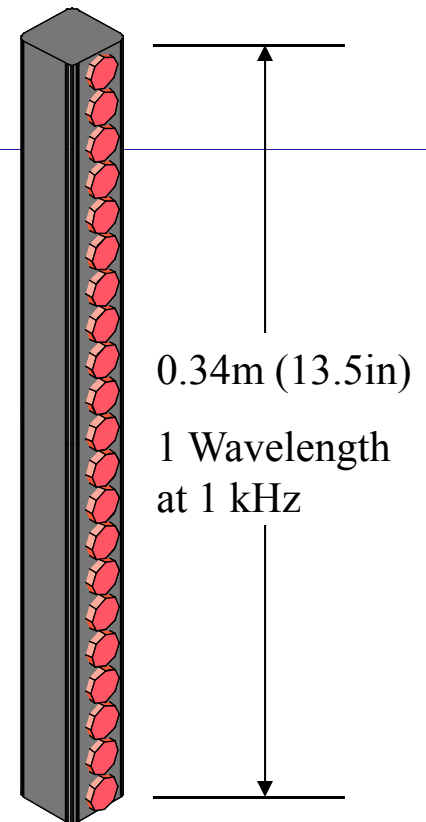
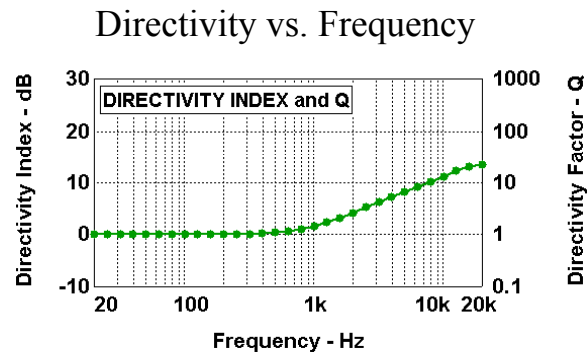
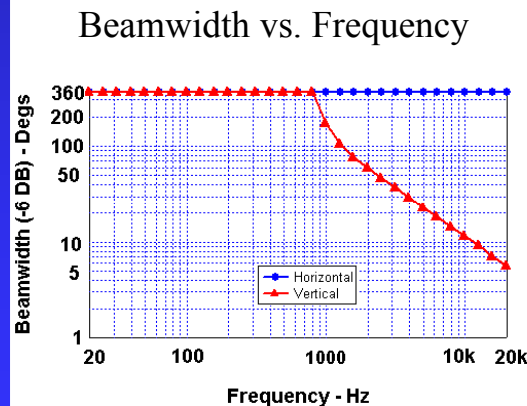
Radius of balloon is proportional to level in dB: 0 dB outside, -40 dB at center. Level is also color coded with a yellow-hot color scheme:



# Straight-Line Array Sound Radiation

Illustrates the directional pattern of a source that exhibits omni-directional radiation in one plane and directional radiation in the other plane.

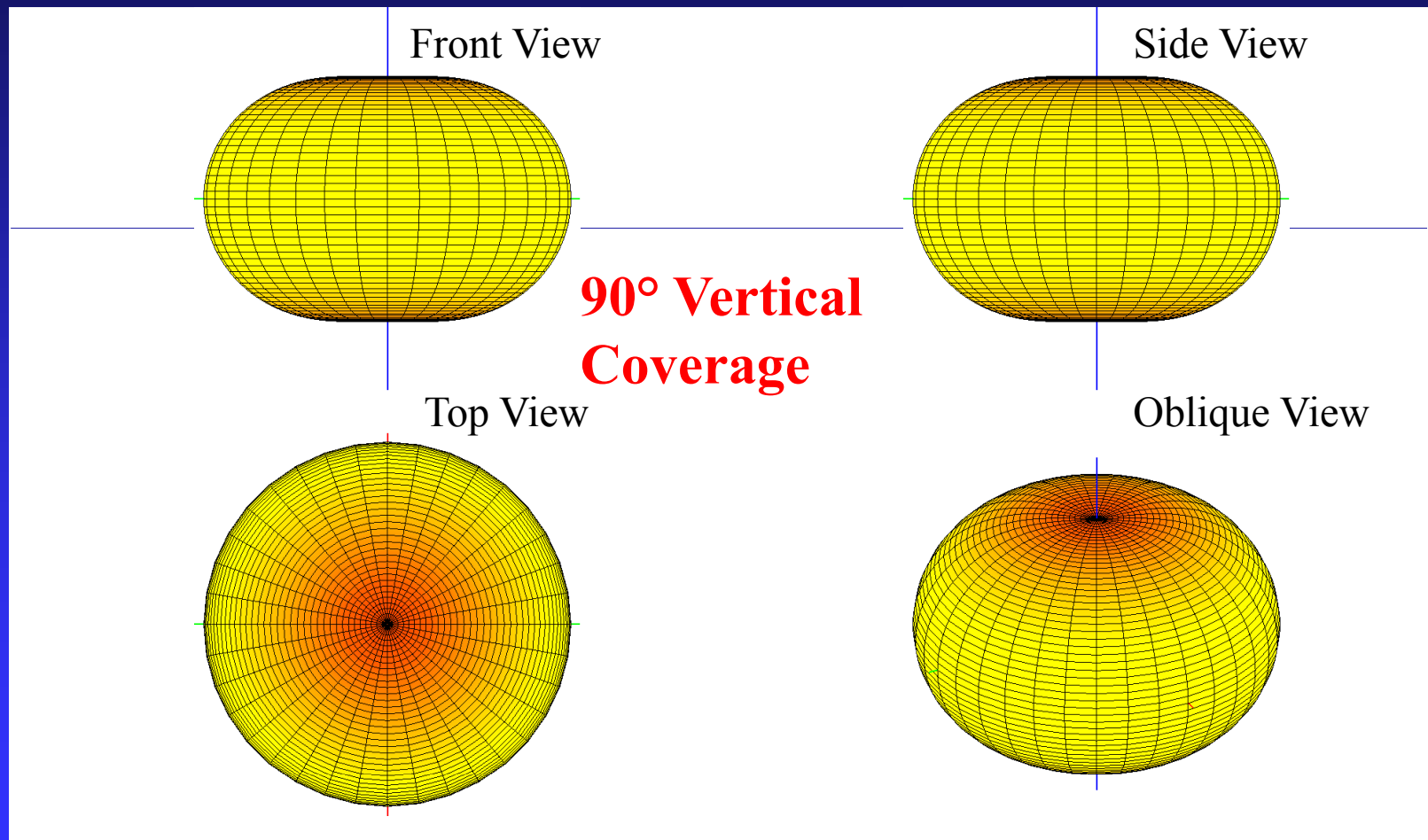
Assume a 50-point source straight-line array with Hann amplitude shading to minimize side lobes.  
(Spacing = 6.8mm (0.27"))



# Straight-Line Array: Cont.

## Broad Vertical Coverage

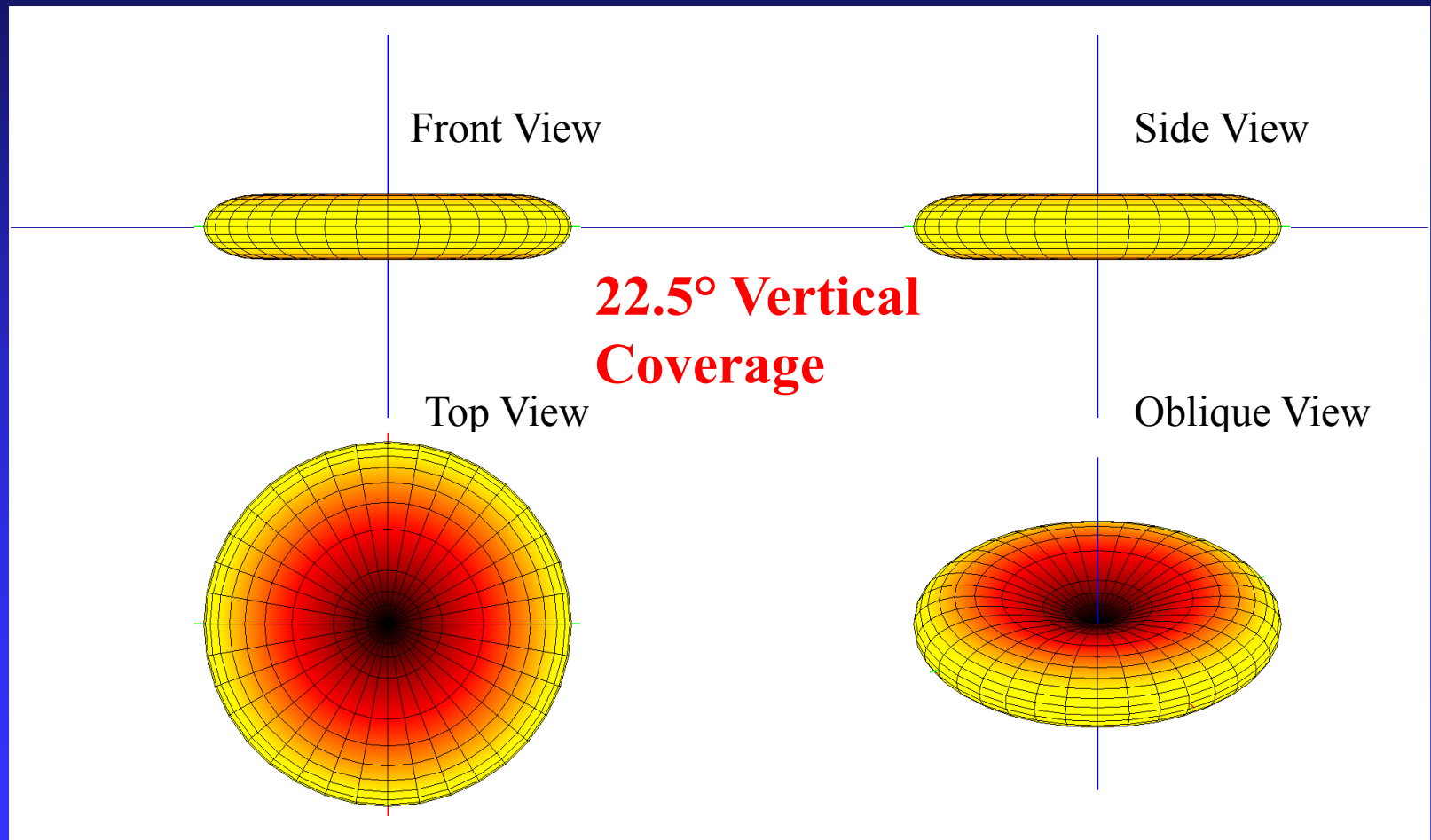
Balloon Plots at 1.4 kHz (Vertical Beamwidth =  $90^\circ$ )



# Straight-Line Array: Cont.

## Narrow Vertical Coverage

Balloon Plots at 5.3 kHz (Vertical Beamwidth =  $22.5^\circ$ )





# Straight-Line Array: Cont.

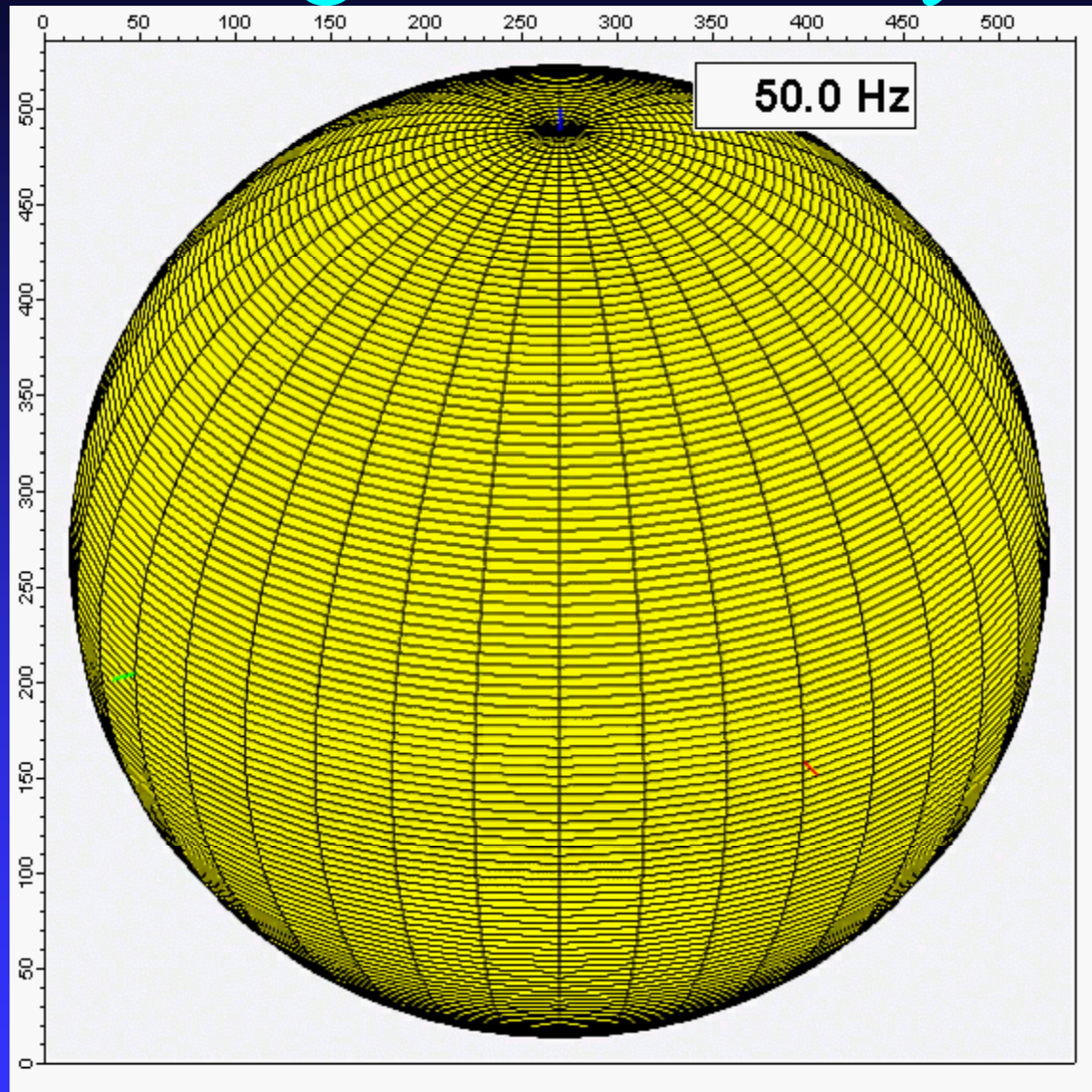
## Play Movie of 3D Radiation of Straight-Line Array

(Oblique View: 50 Hz to 20 kHz at one-third-octave intervals)

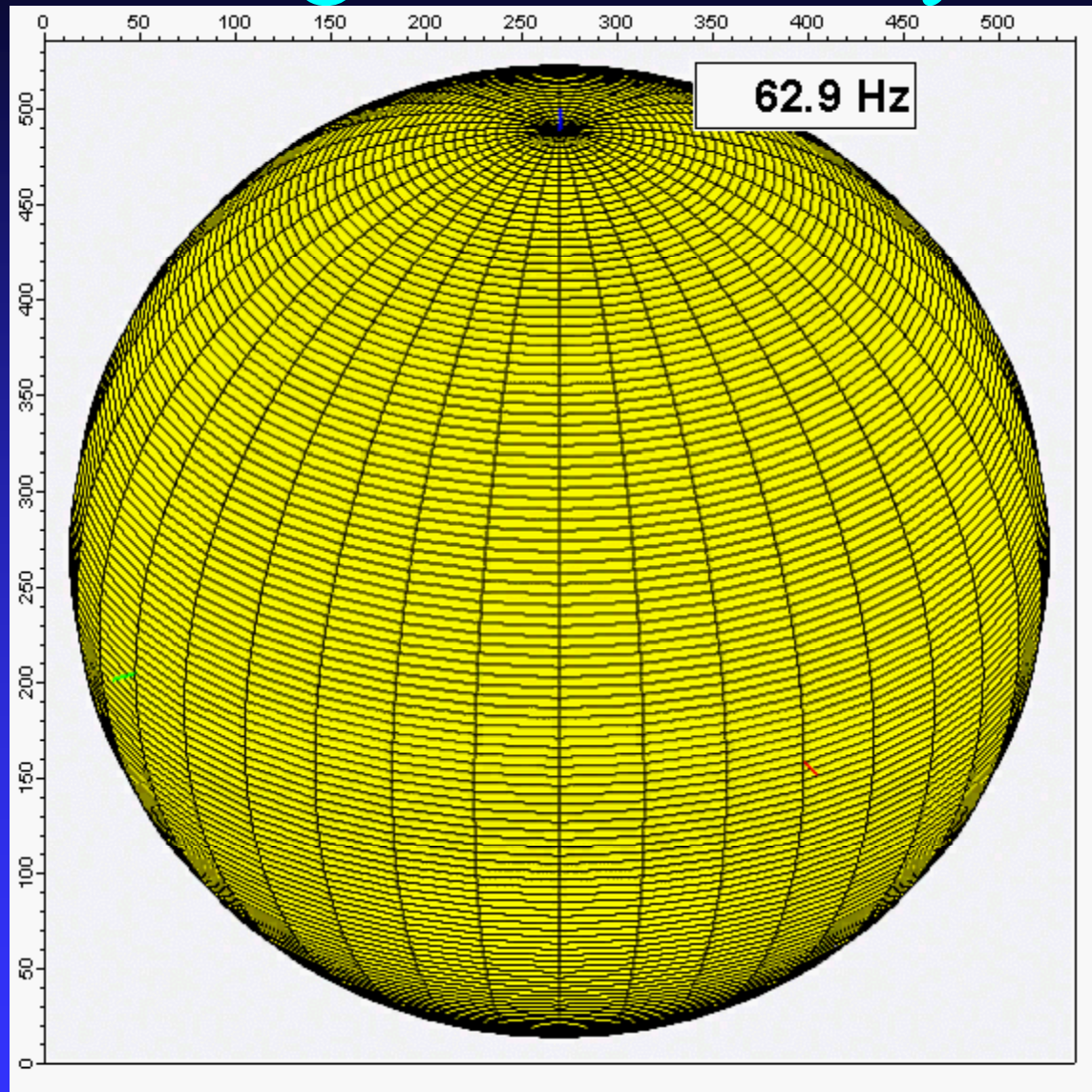
# Start of Movie

(Slides 18-43)

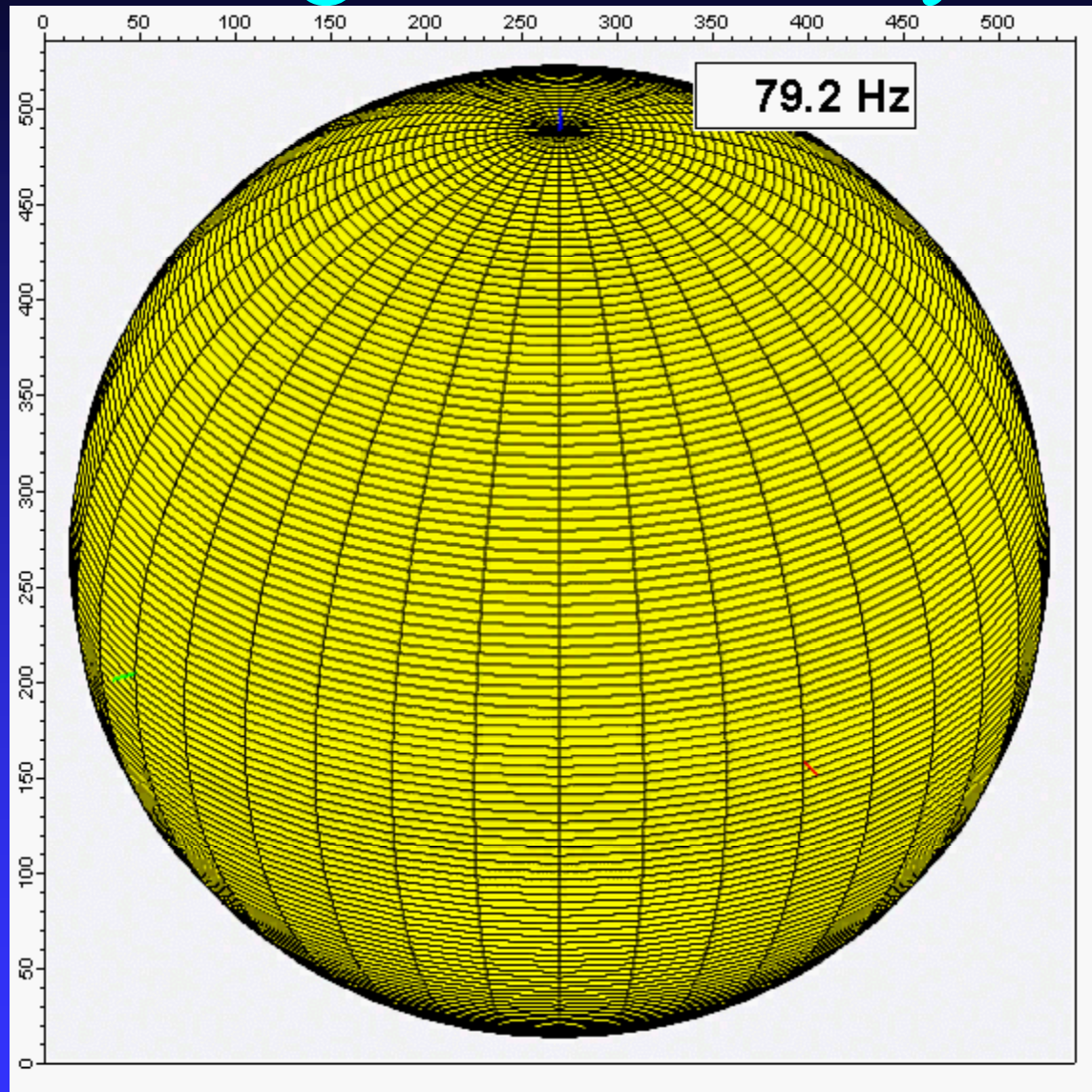
# Straight-Line Array: Cont.



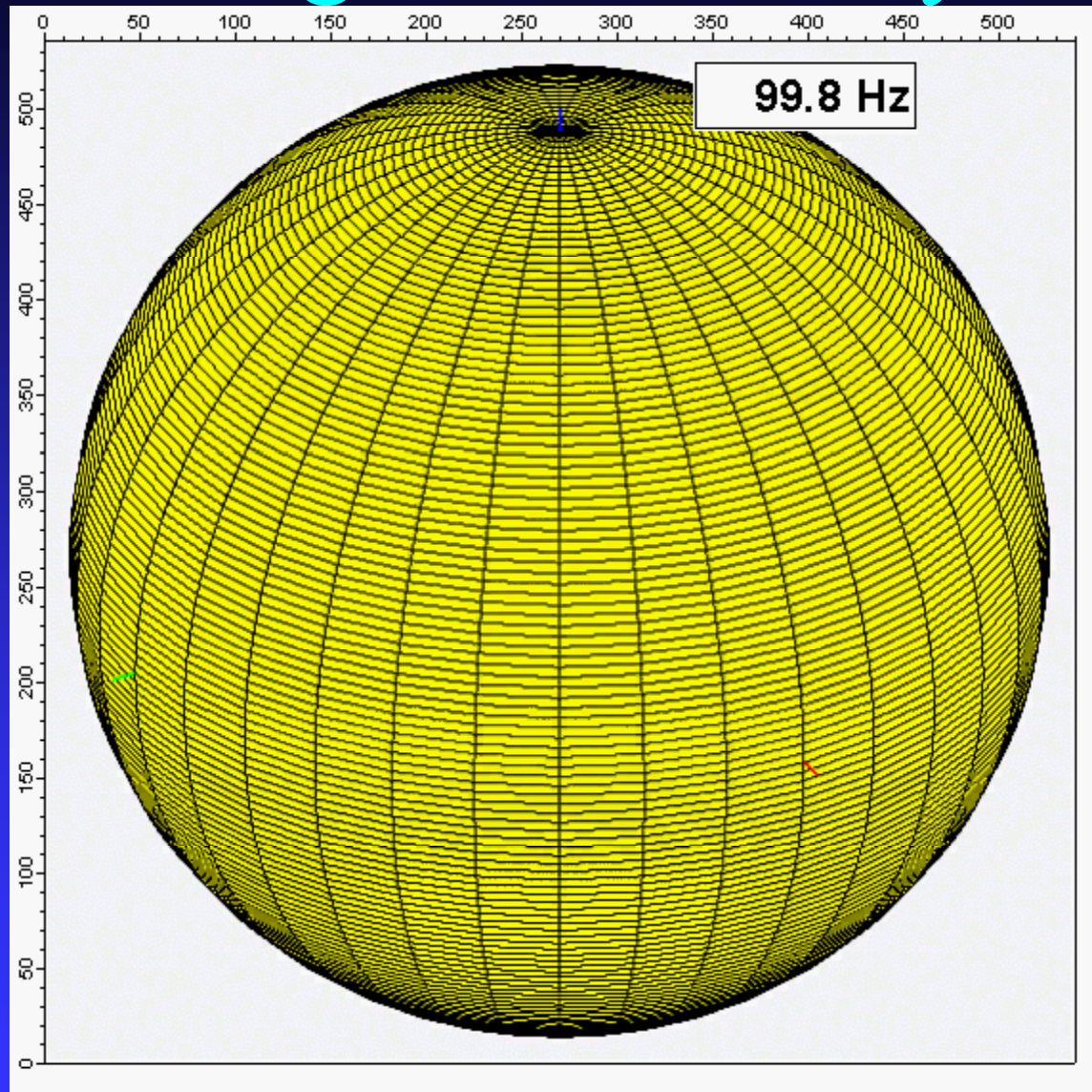
# Straight-Line Array: Cont.



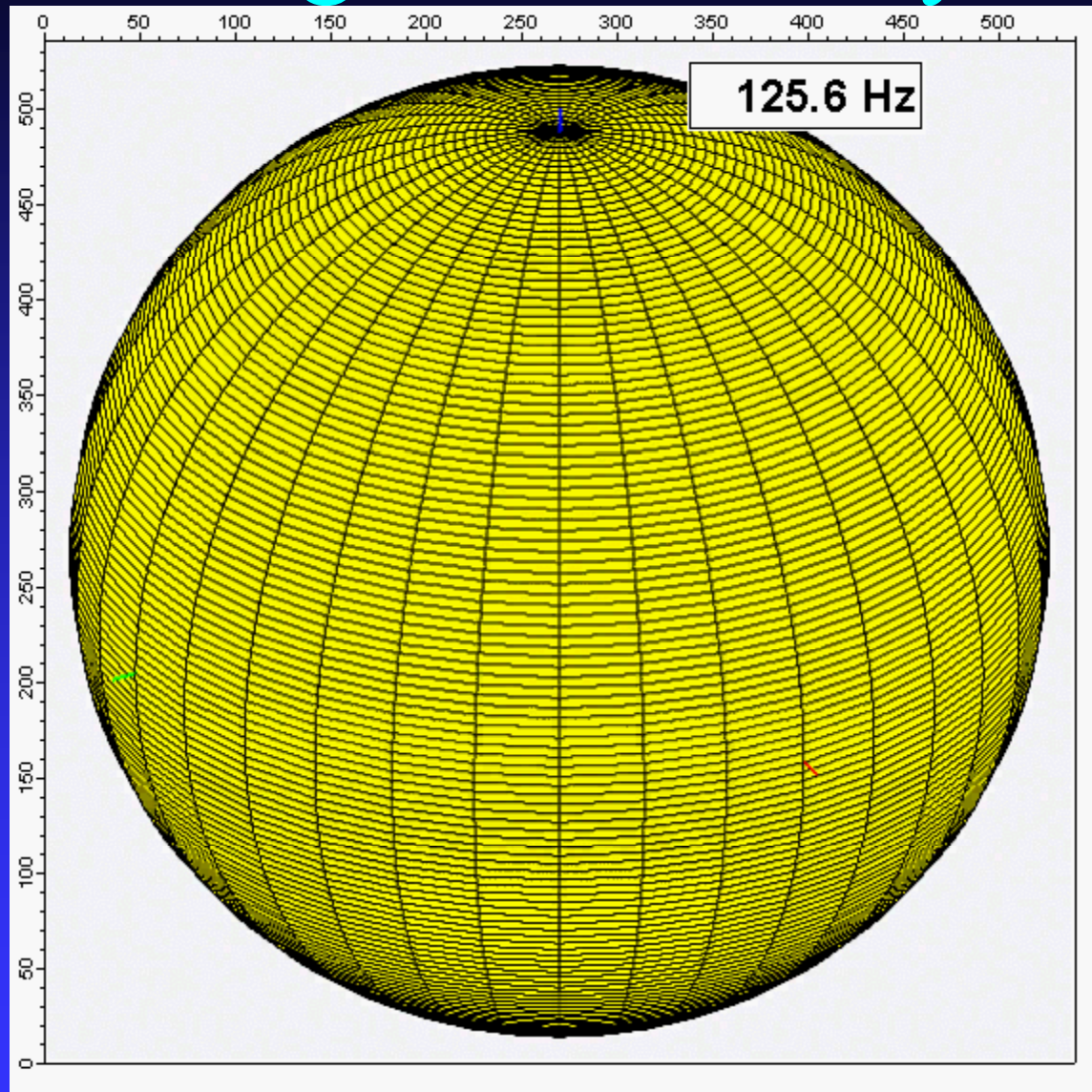
# Straight-Line Array: Cont.



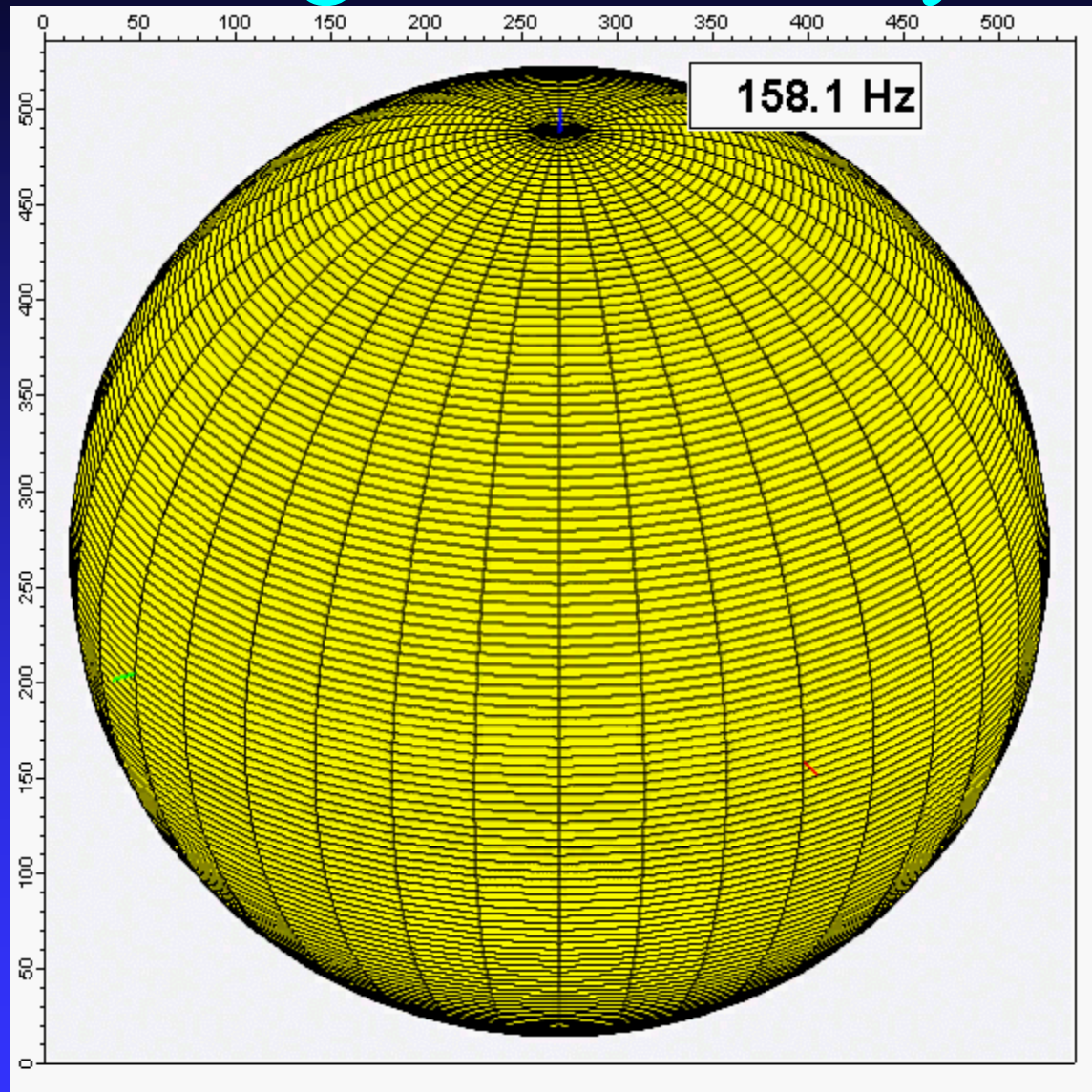
# Straight-Line Array: Cont.



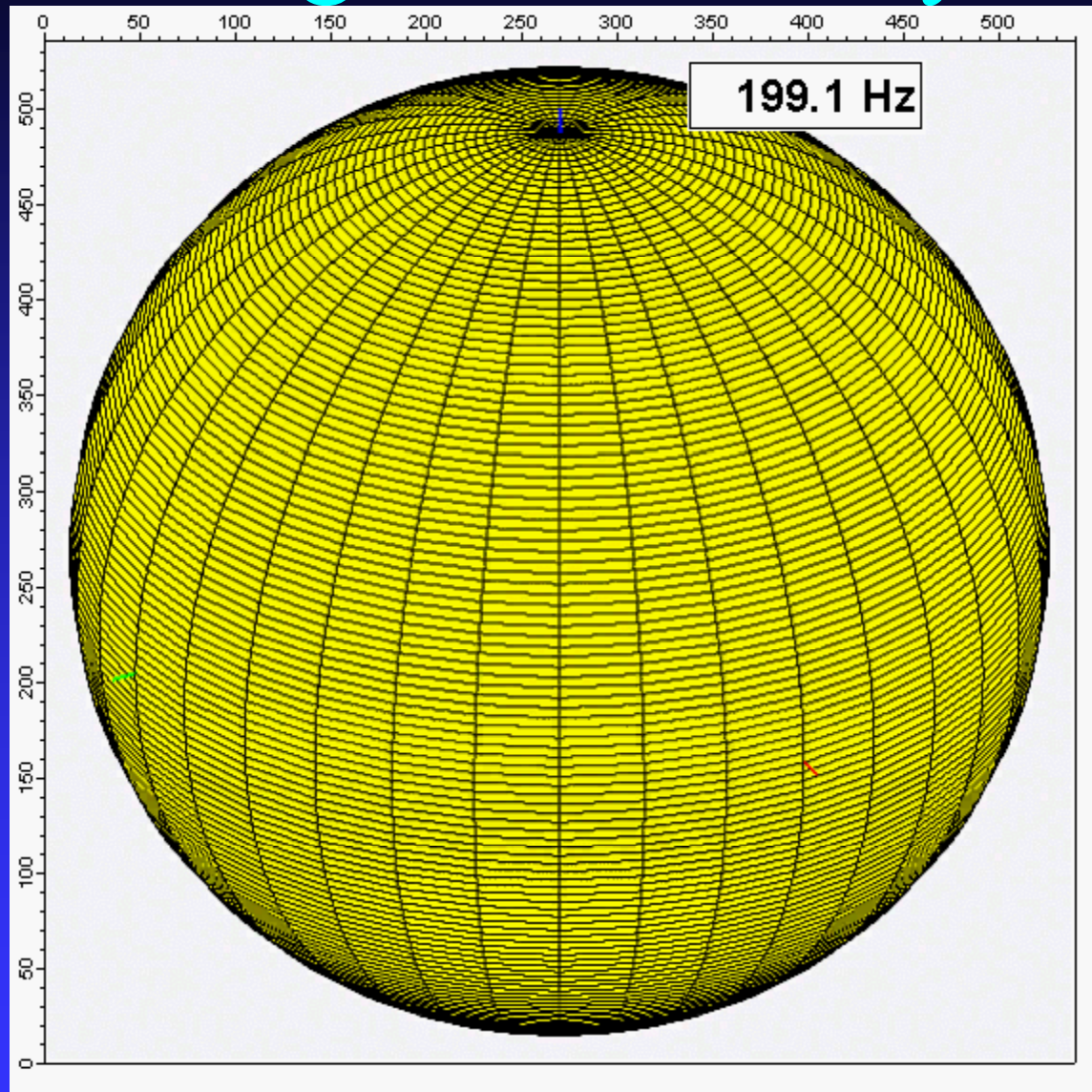
# Straight-Line Array: Cont.



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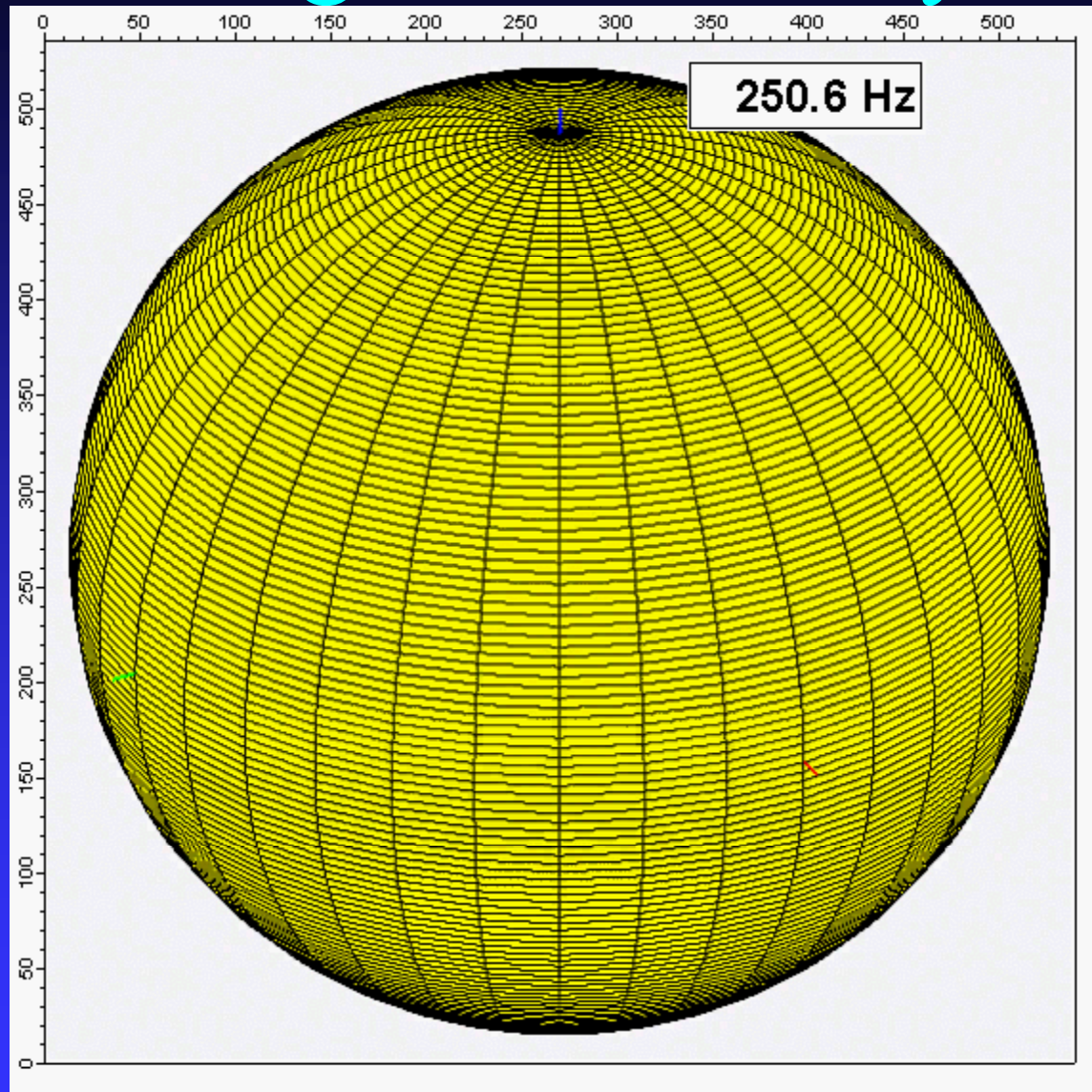


# Straight-Line Array: Cont.

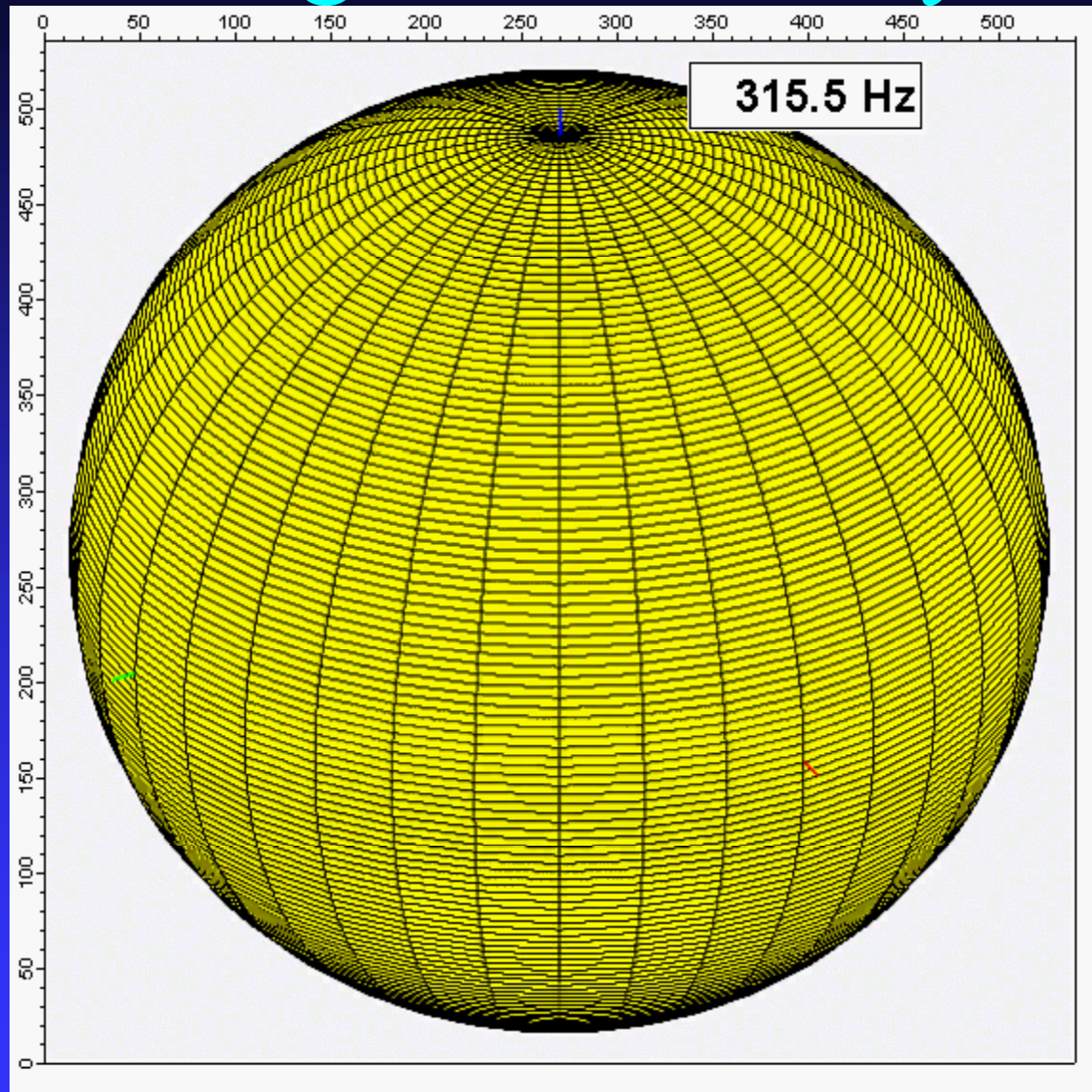




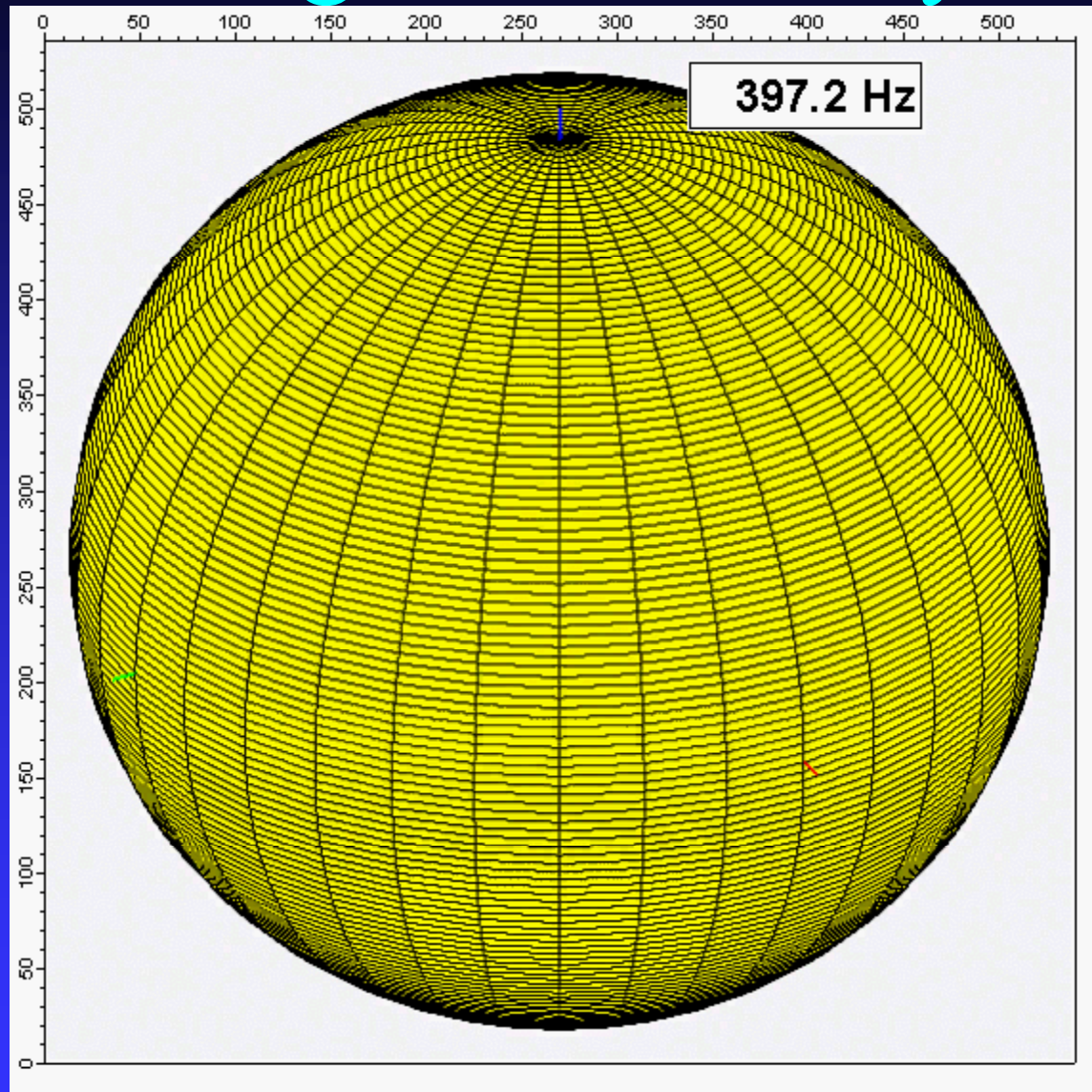
# Straight-Line Array: Cont.



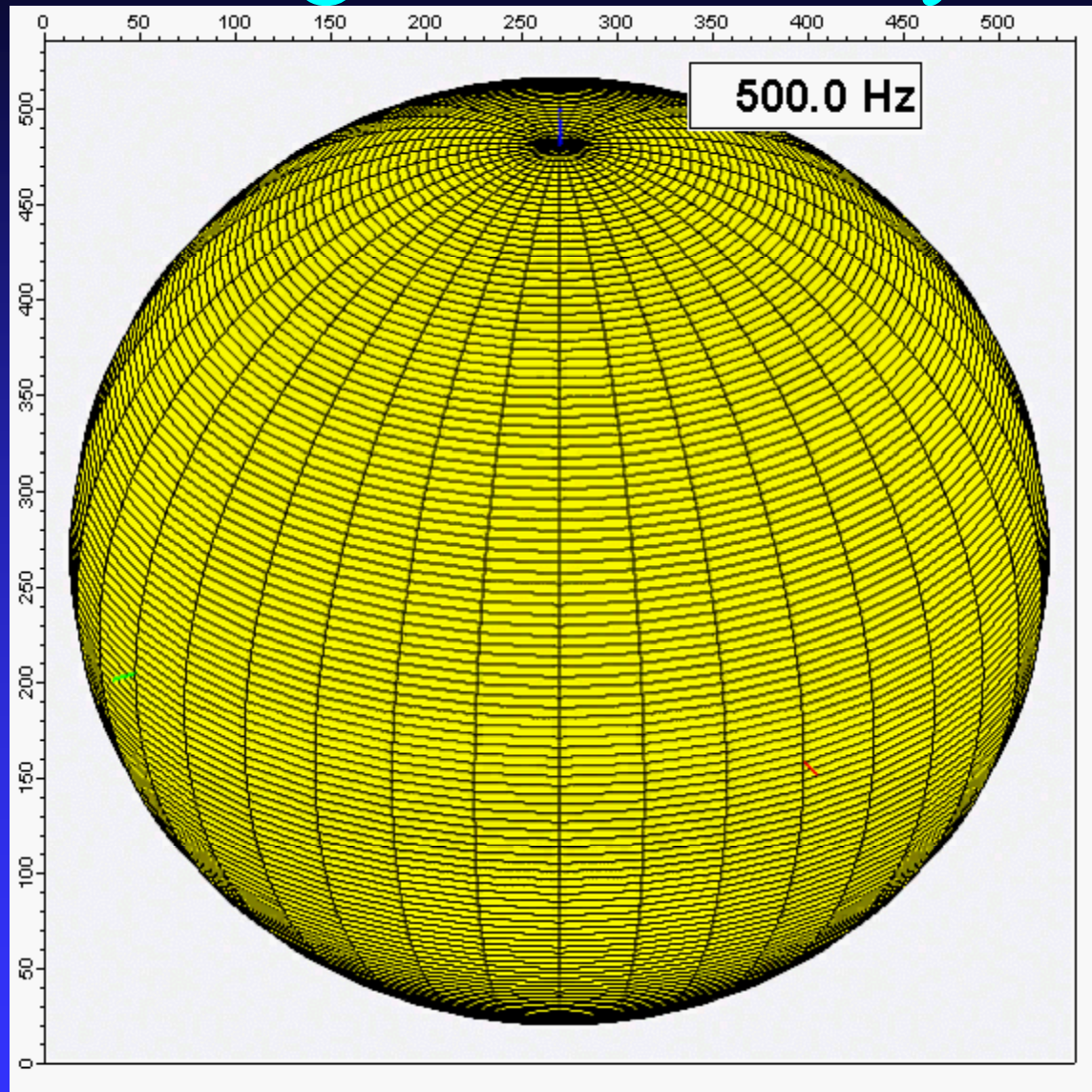
# Straight-Line Array: Cont.



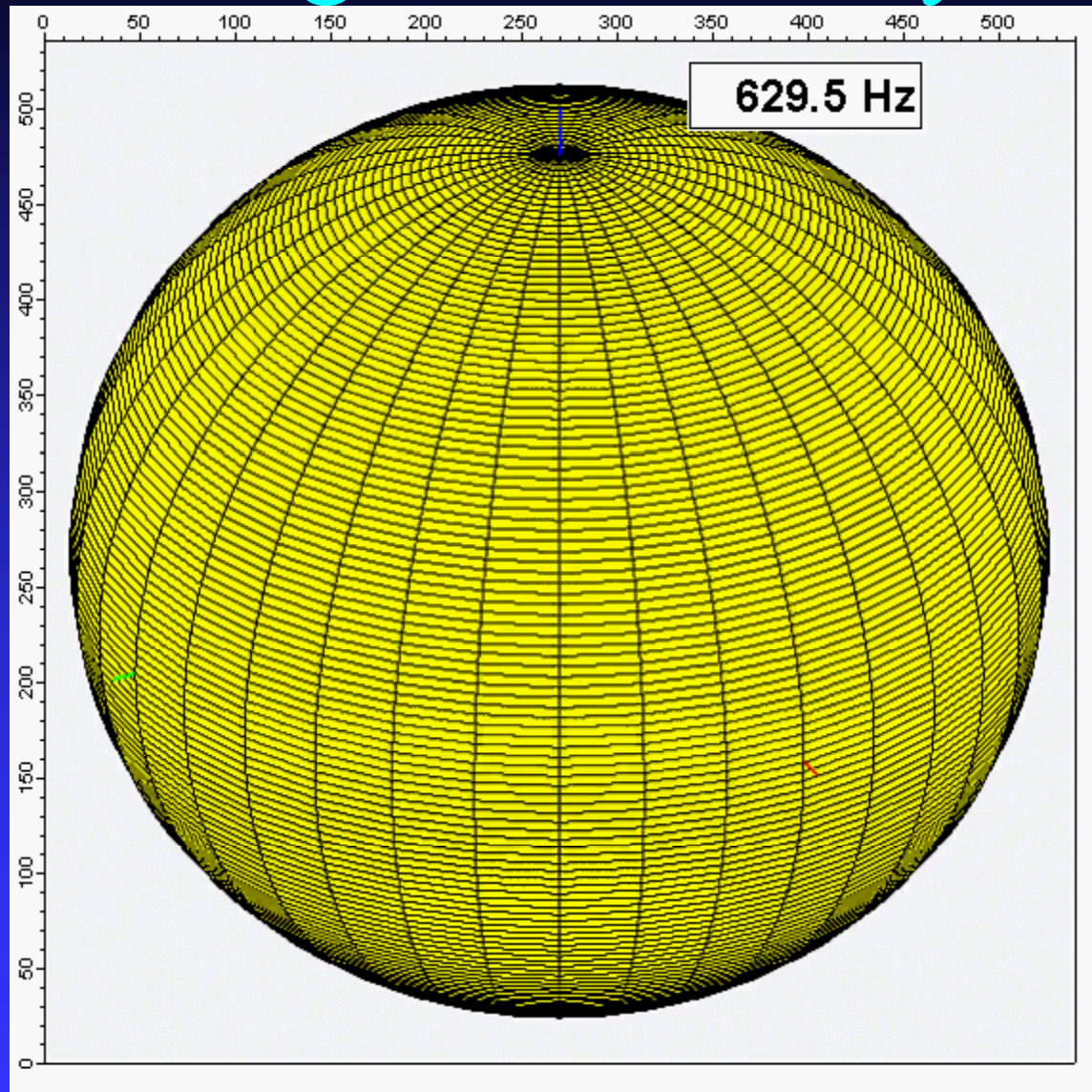
# Straight-Line Array: Cont.



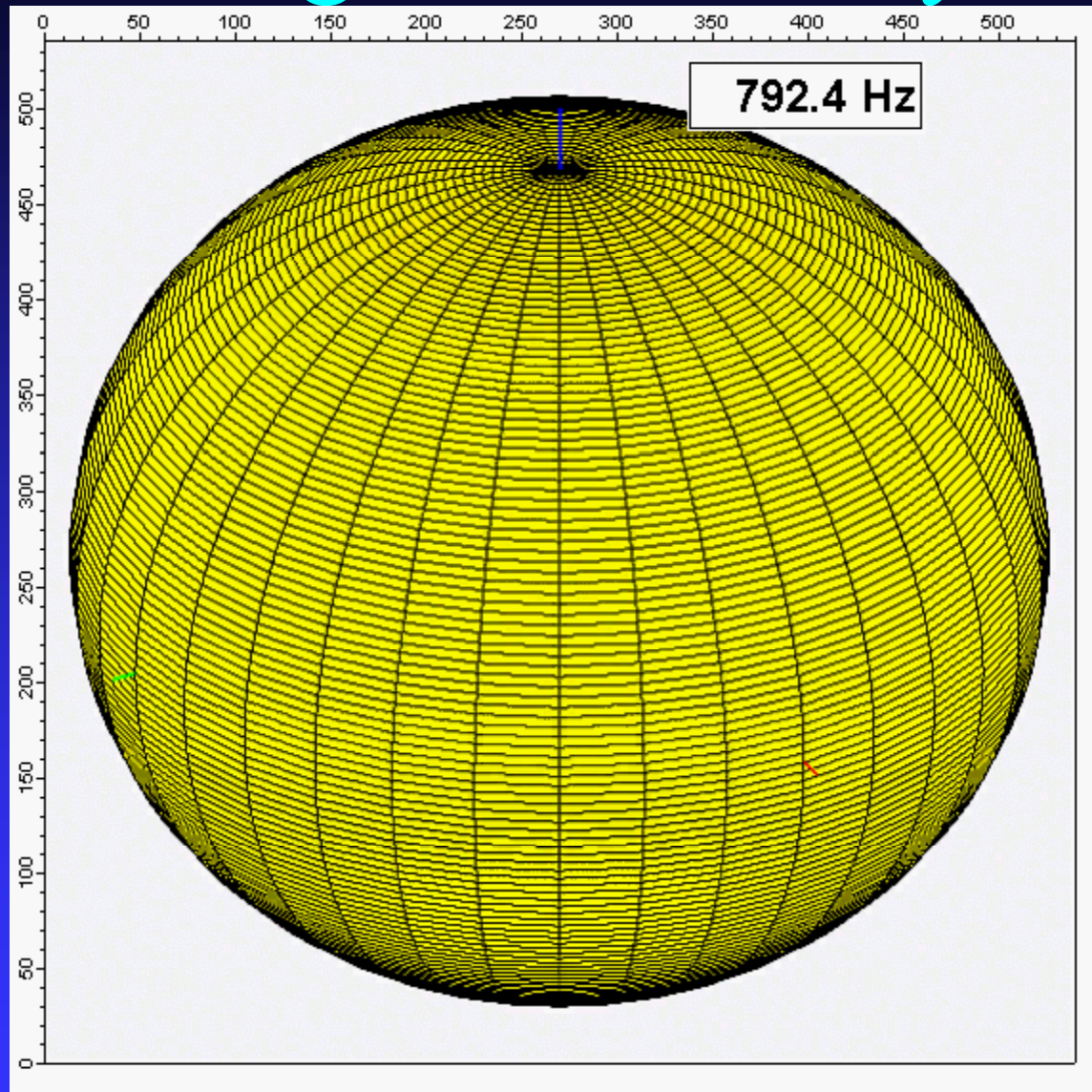
# Straight-Line Array: Cont.



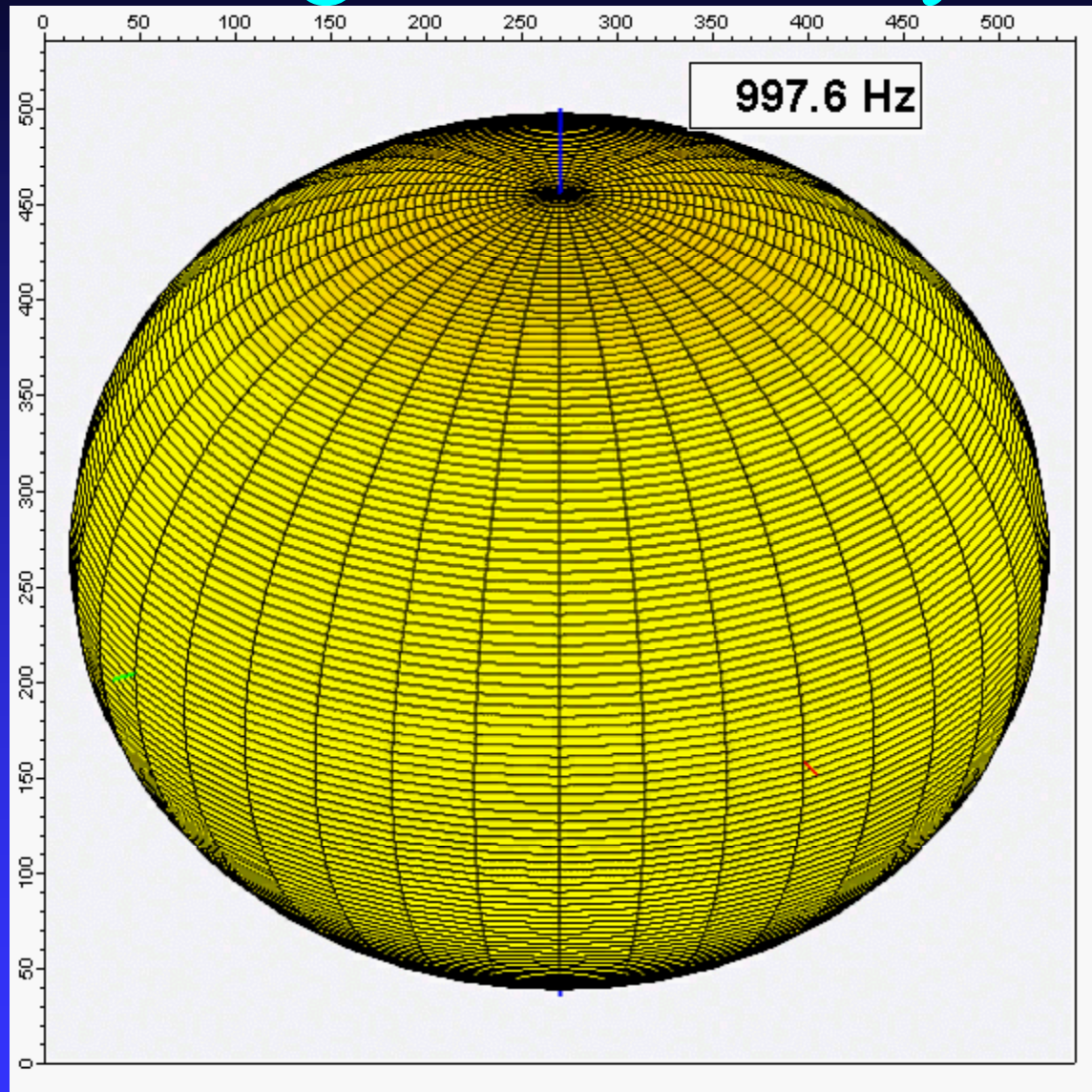
# Straight-Line Array: Cont.



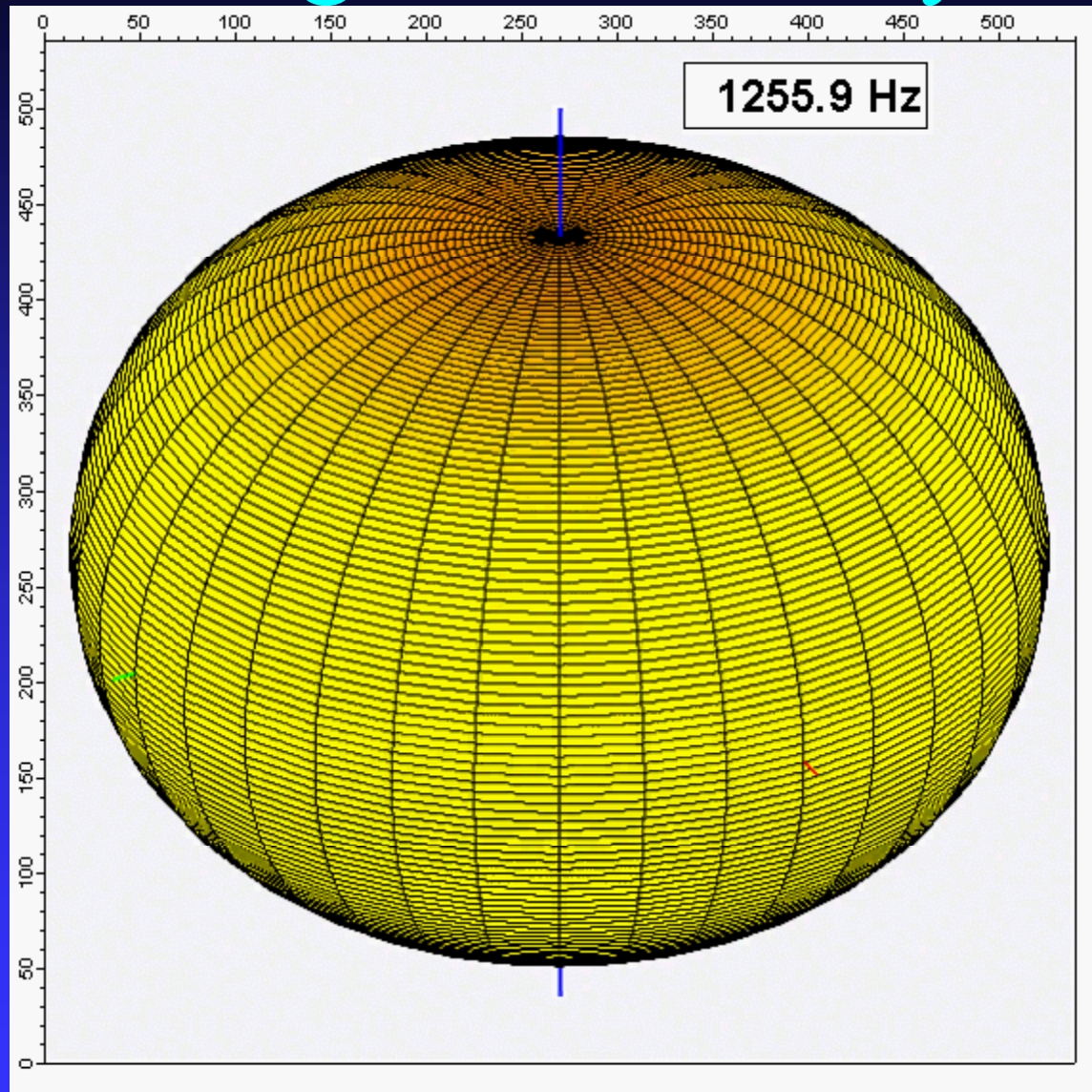
# Straight-Line Array: Cont.



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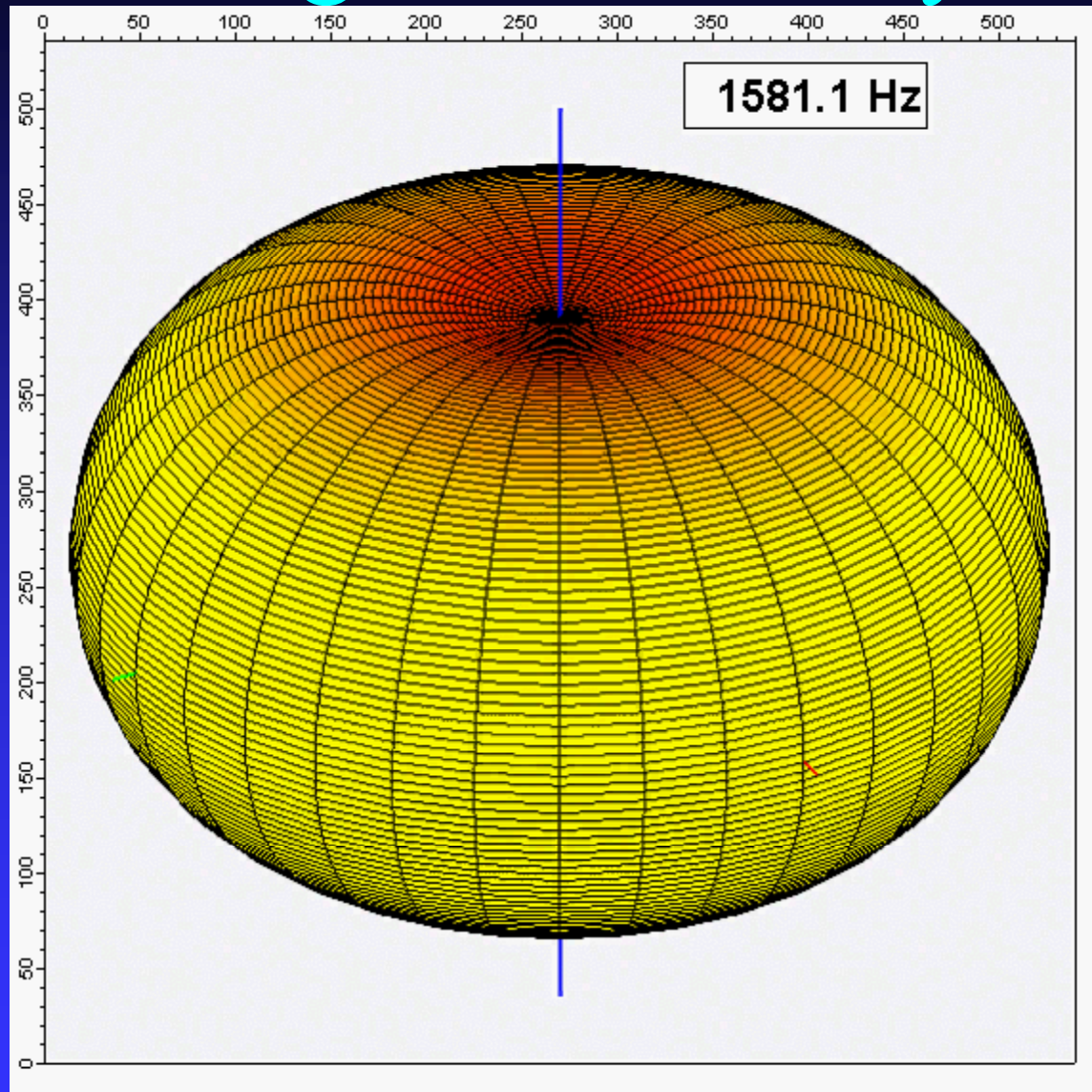


# Straight-Line Array: Cont.

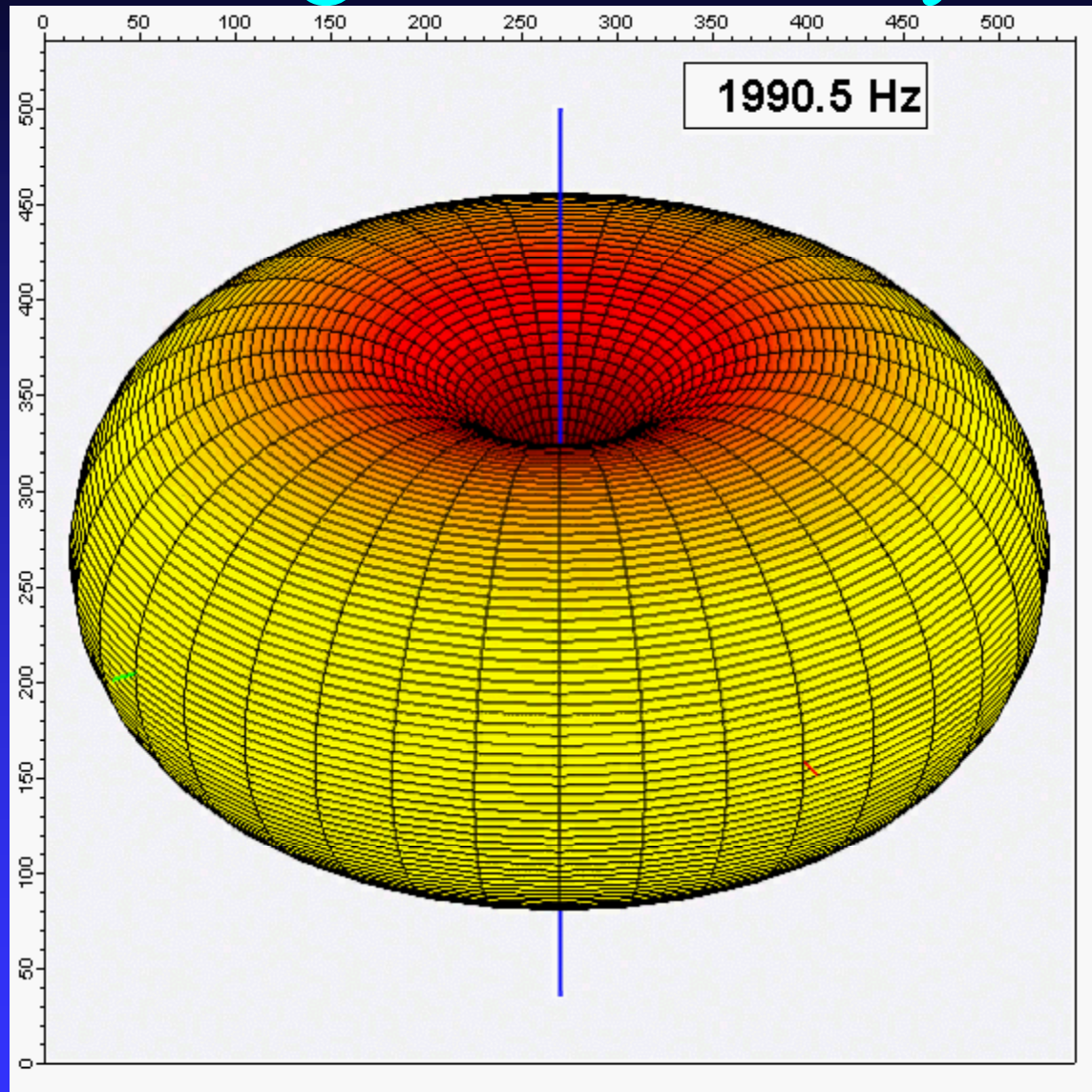




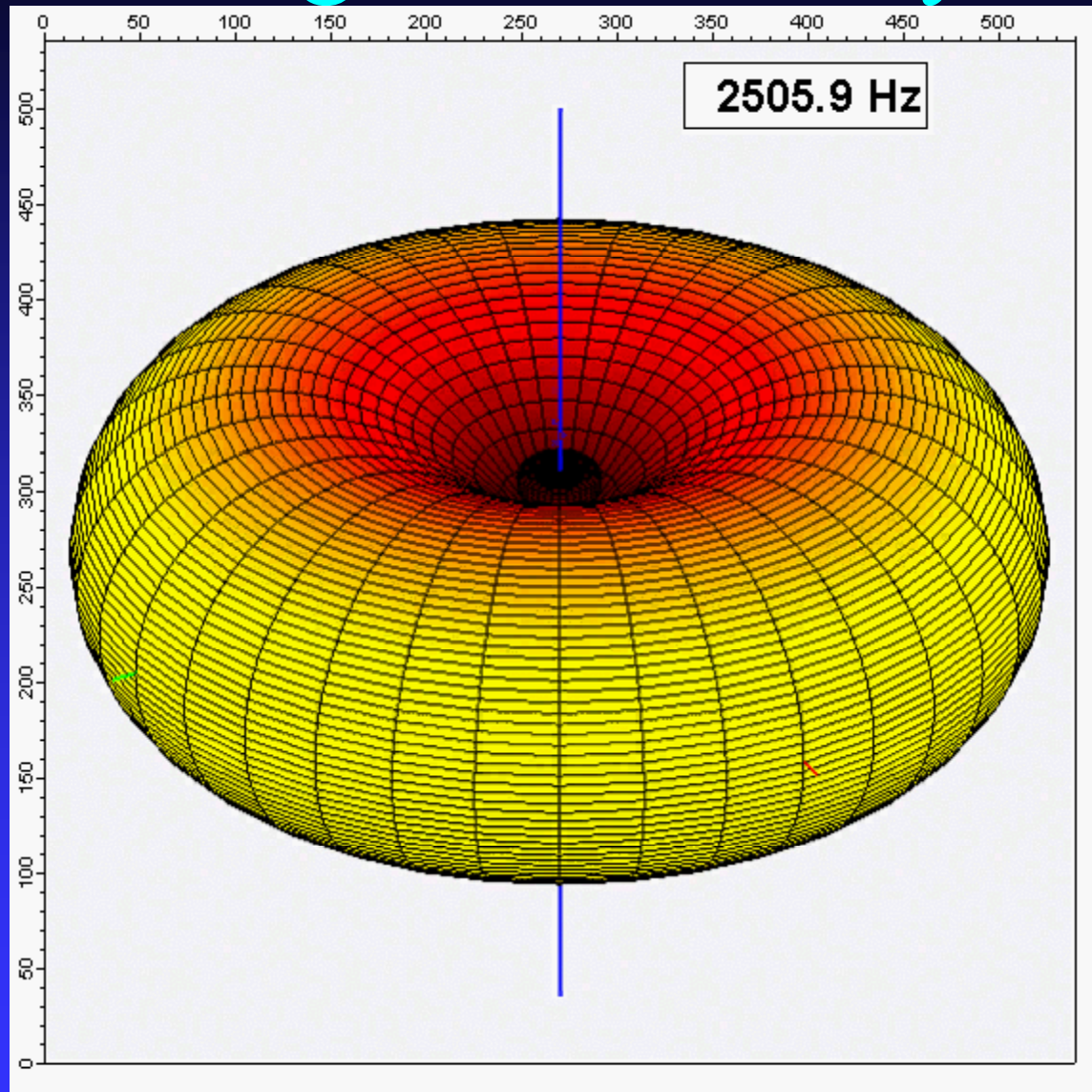
# Straight-Line Array: Cont.



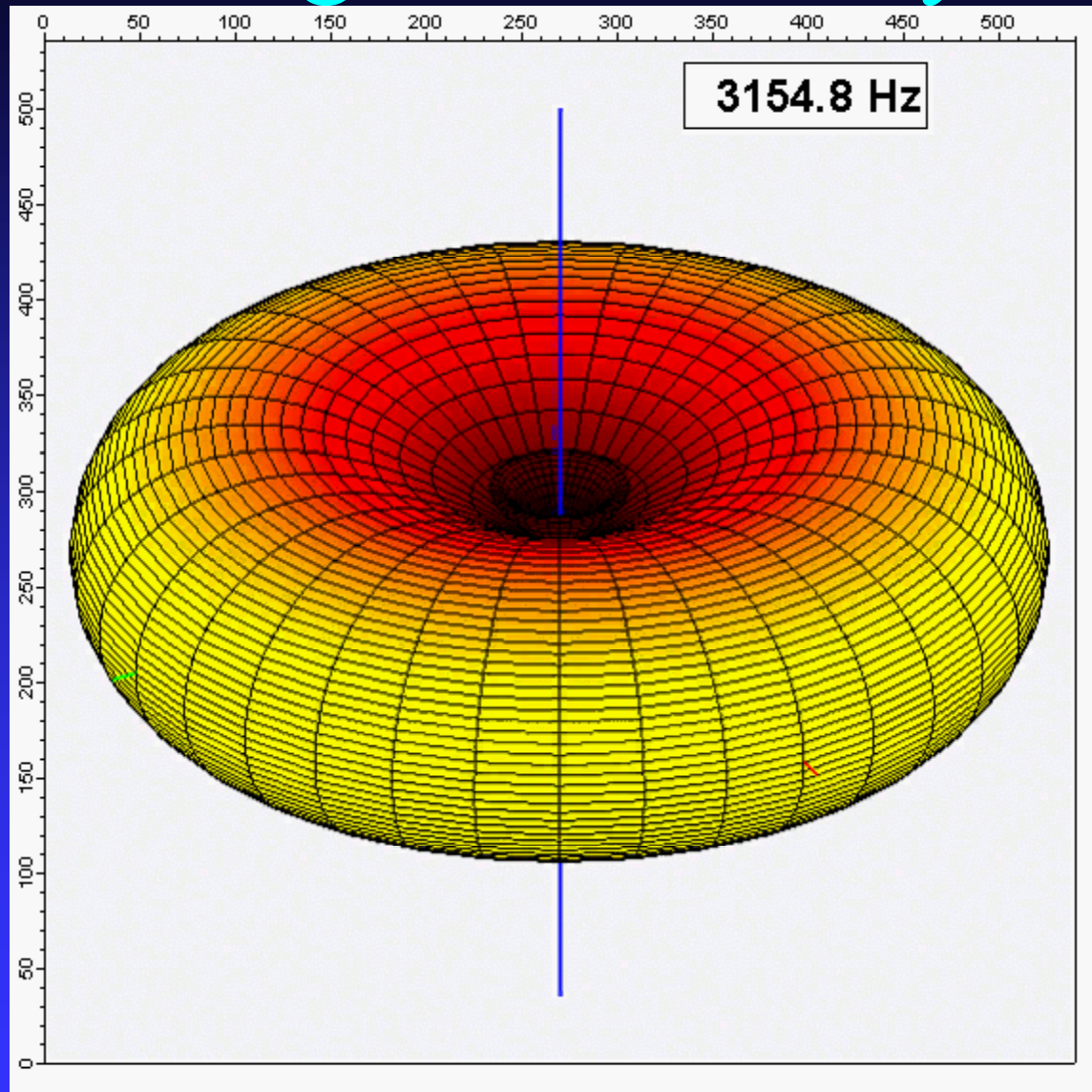
# Straight-Line Array: Cont.



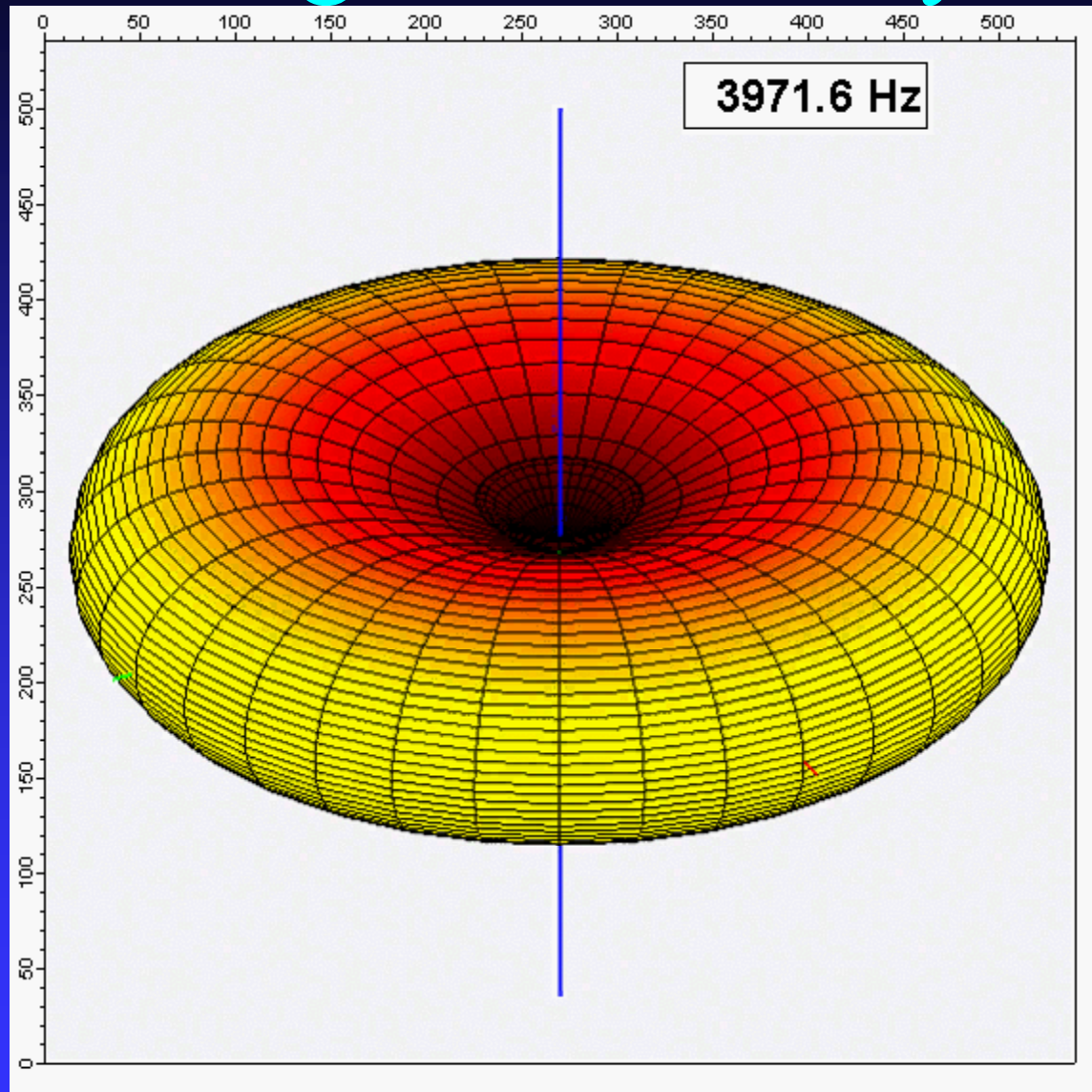
# Straight-Line Array: Cont.



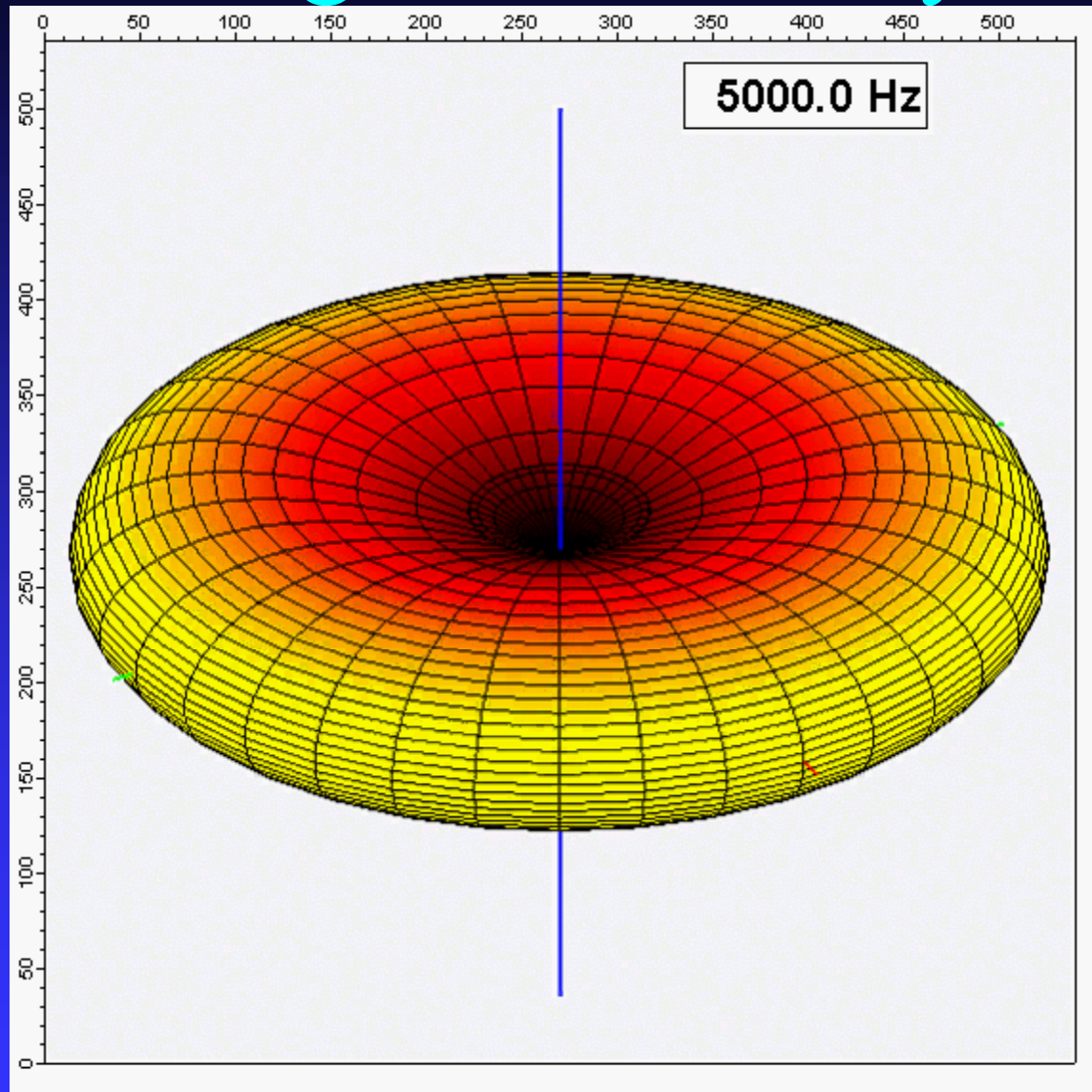
# Straight-Line Array: Cont.



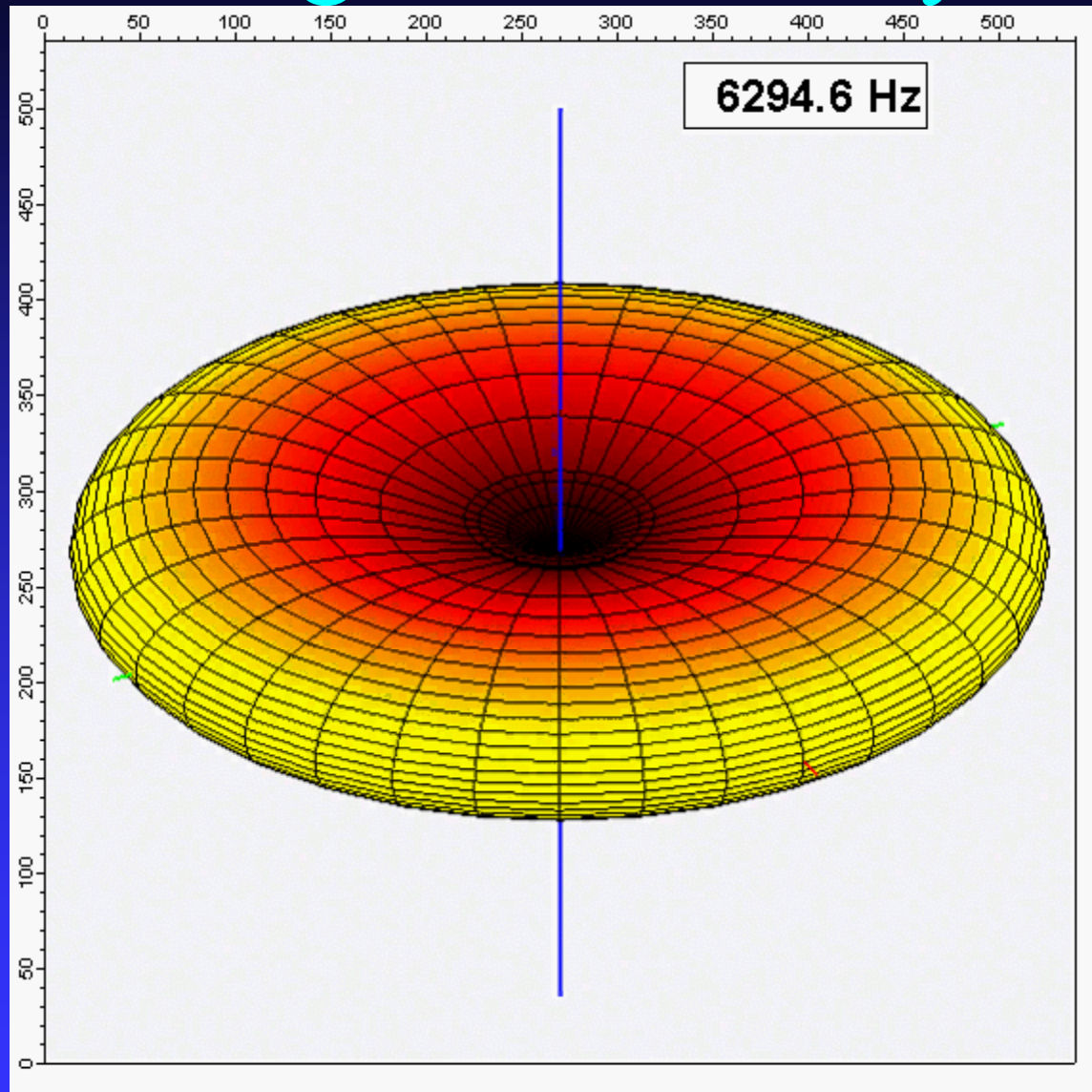
# Straight-Line Array: Cont.



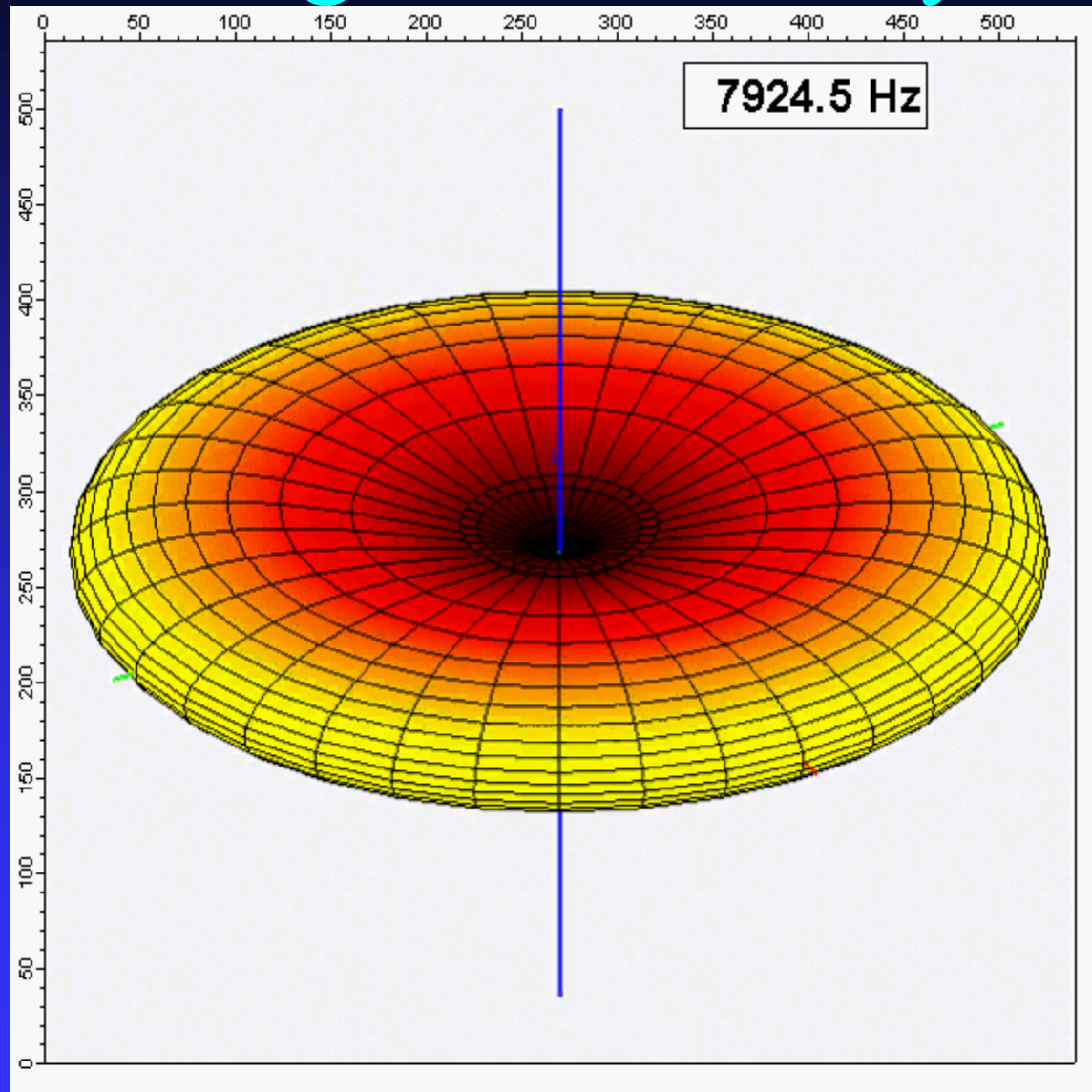
# Straight-Line Array: Cont.



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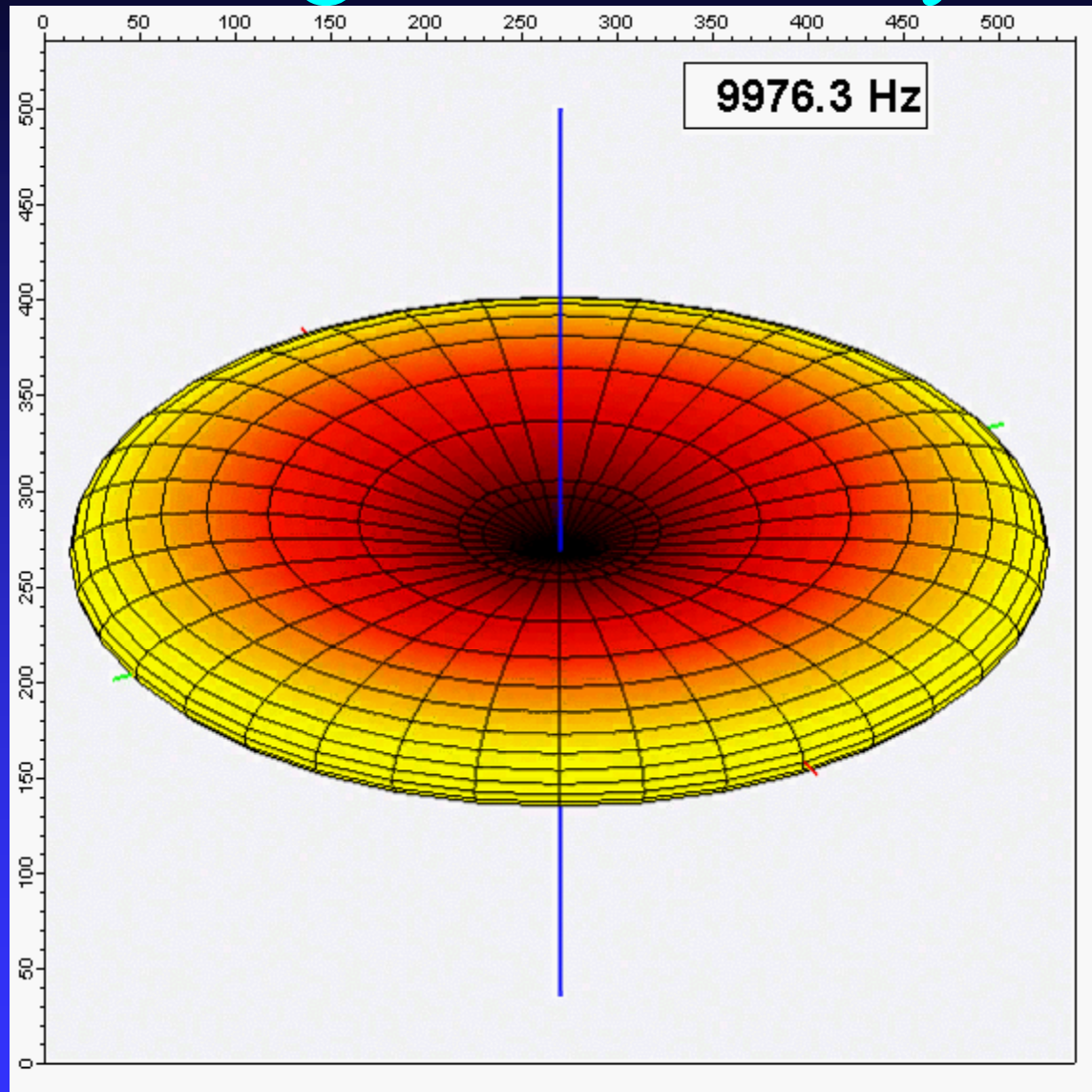


# Straight-Line Array: Cont.

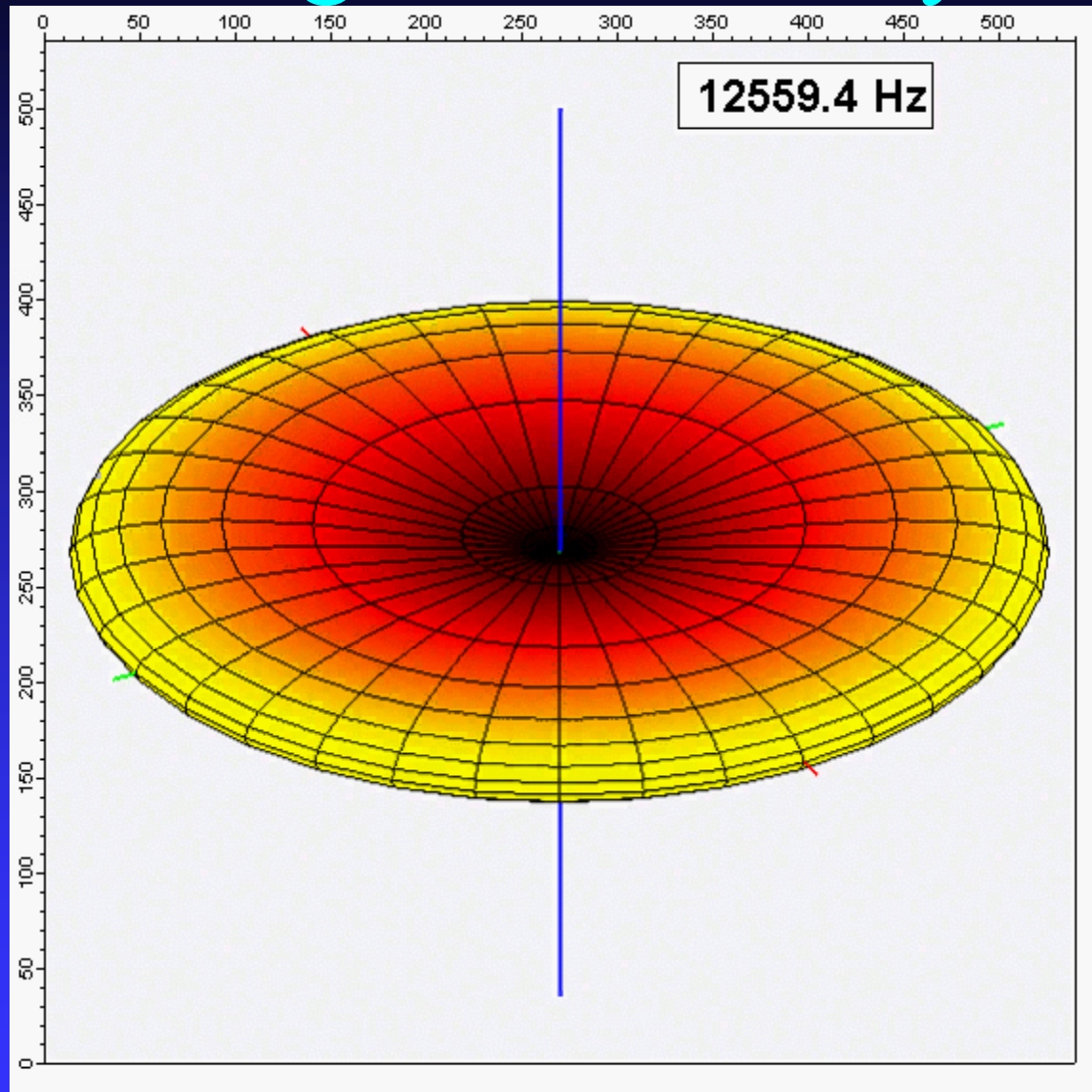




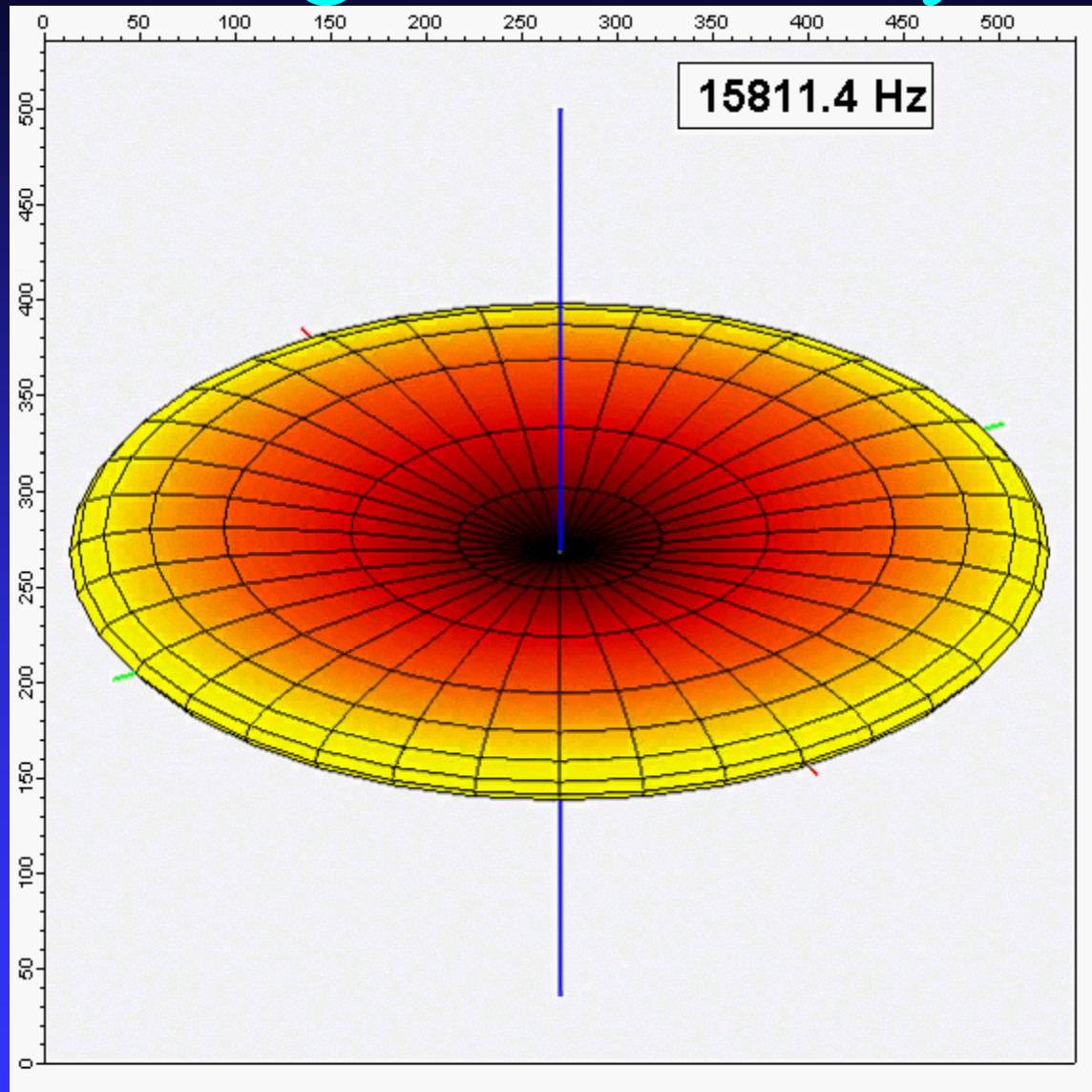
# Straight-Line Array: Cont.



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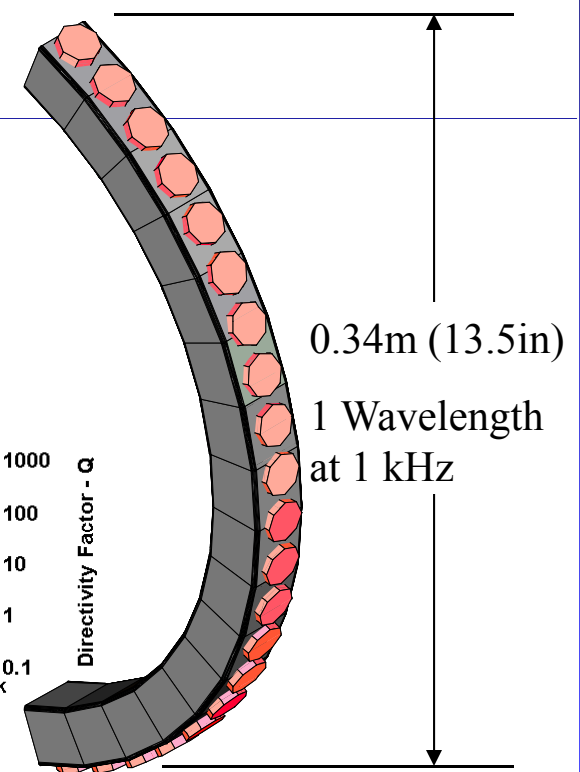
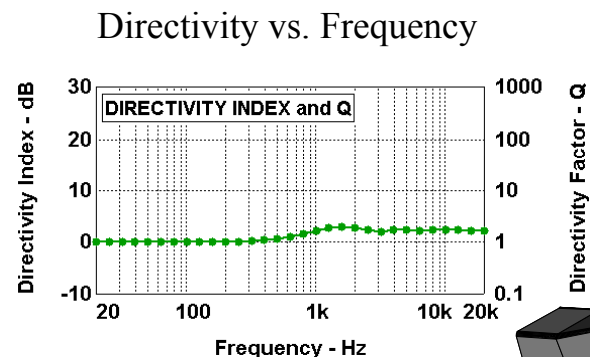
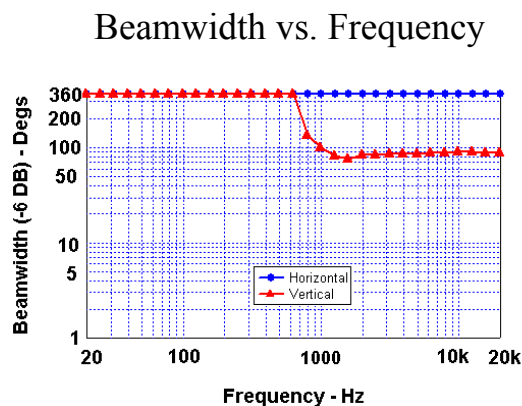
# Straight-Line Array: Cont.

End of  
Straight-Line  
Array Movie

# CBT Curved-Line Array

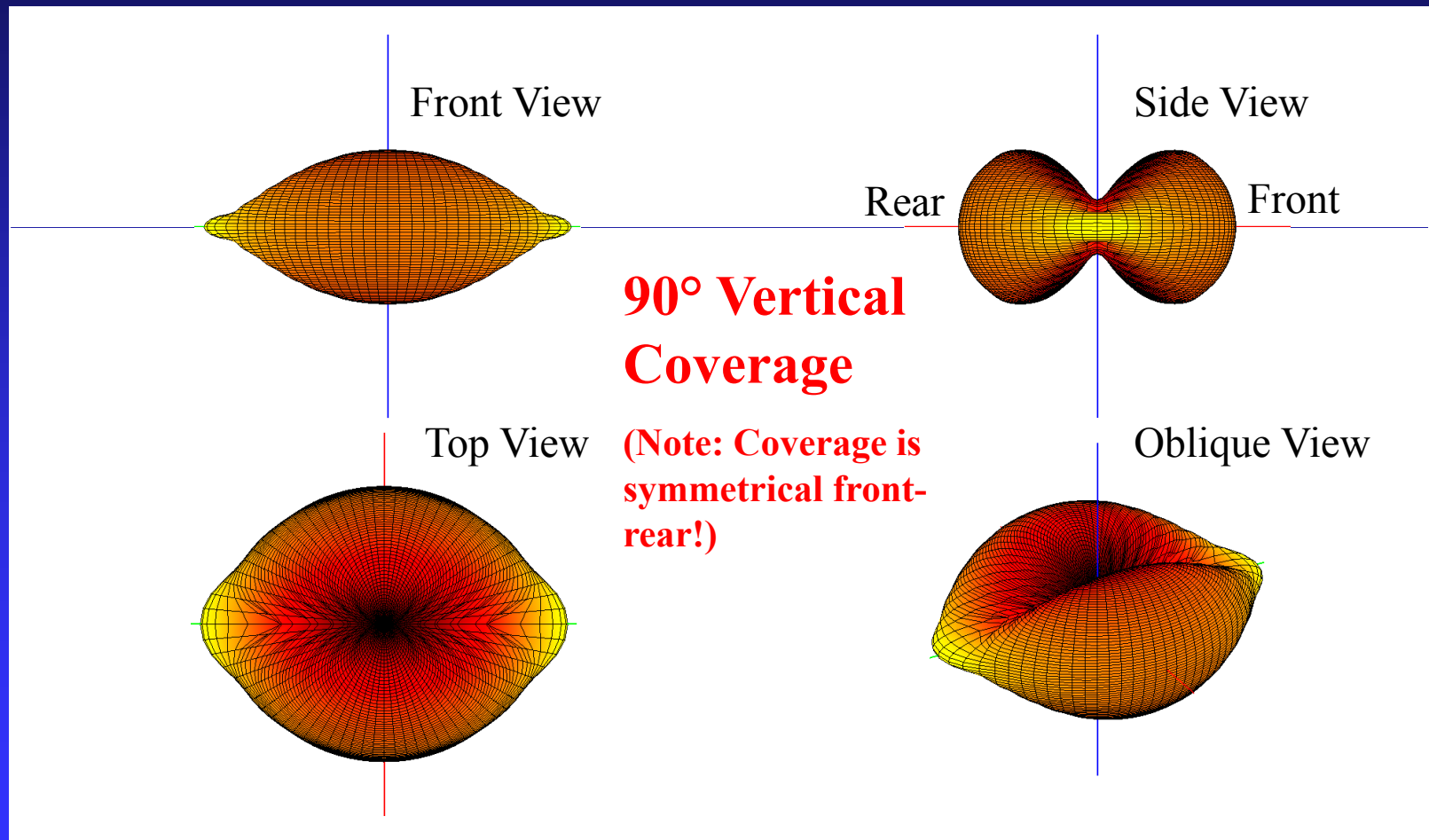
## Sound Radiation: Broad Vertical Coverage

Assume a 50-point source CBT 140° curved-line array with Legendre amplitude shading providing a 90° vertical coverage above 1 kHz.



# CBT Curved-Line Array: Cont. Broad Vertical Coverage

Balloon Plots at 8 kHz (Vertical Beamwidth = 90°)



## CBT Curved Array: Broad Vertical Coverage Cont.

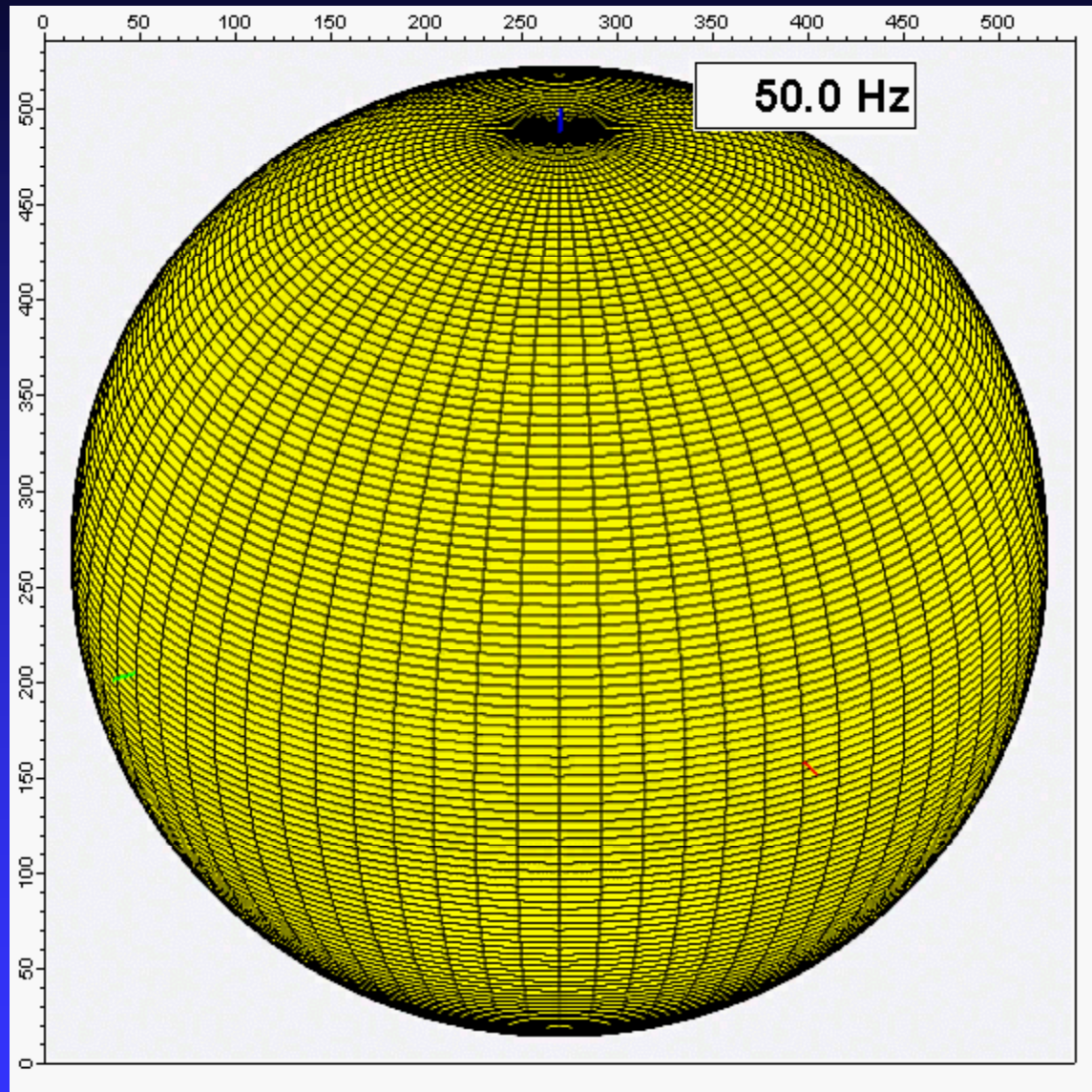
# Play Movie of 3D Radiation of Broad Vertical Coverage CBT Line Array

(Oblique View Normalized to On Axis: 50 Hz to  
16 kHz at one-third-octave intervals)

# Start of Movie

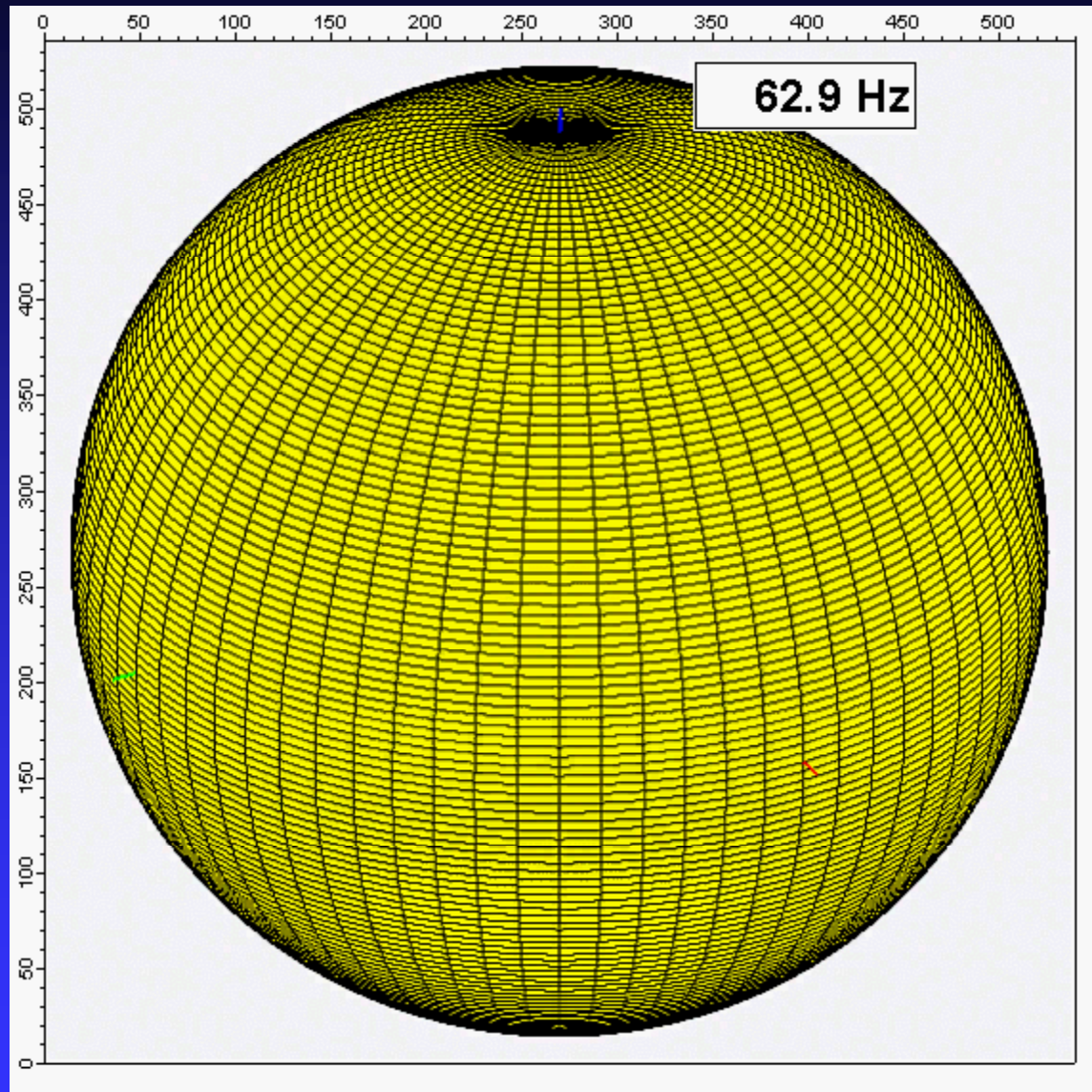
(Slides 48-73)

# CBT Curved Array: Broad Vertical Coverage Cont.

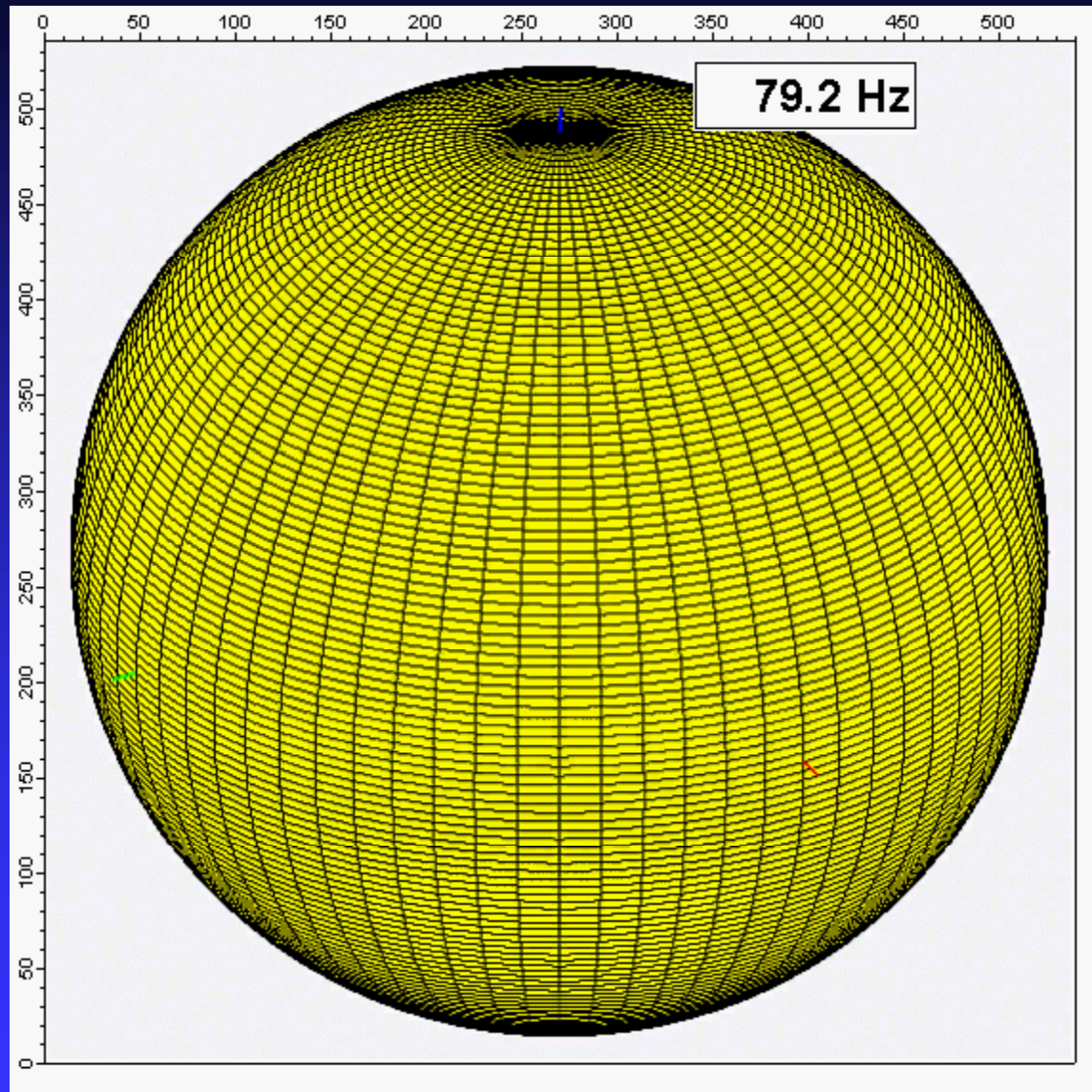




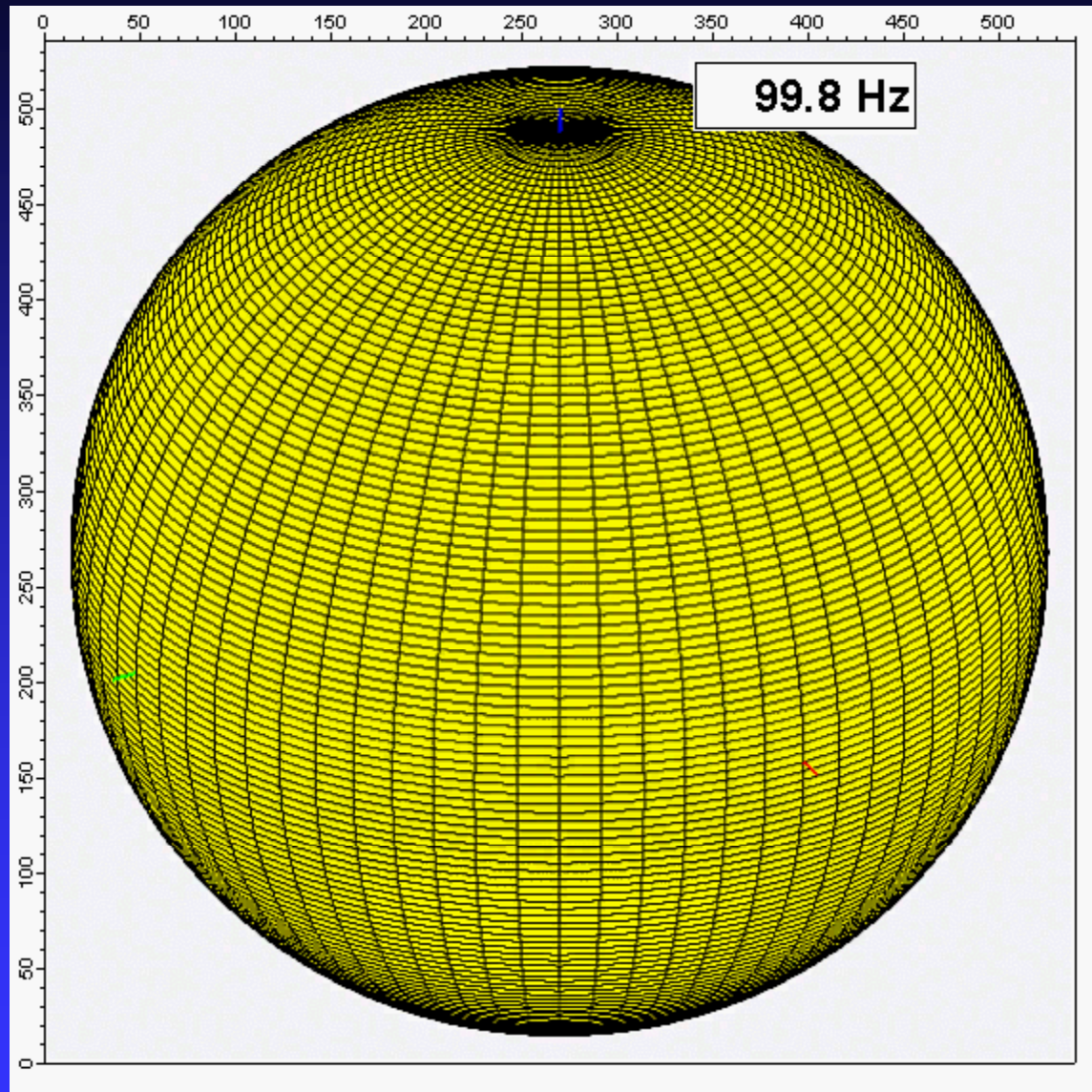
# CBT Curved Array: Broad Vertical Coverage Cont.



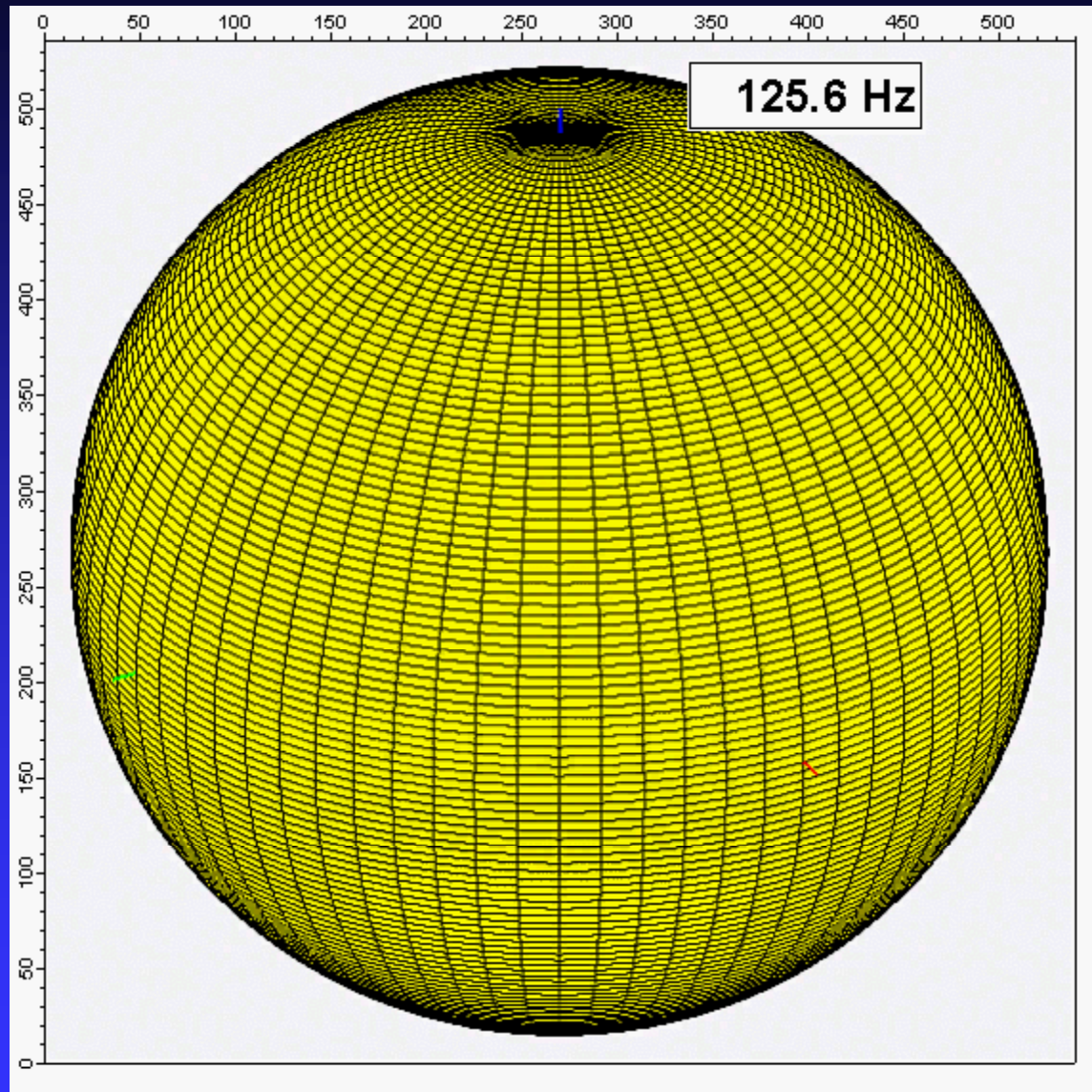
# CBT Curved Array: Broad Vertical Coverage Cont.



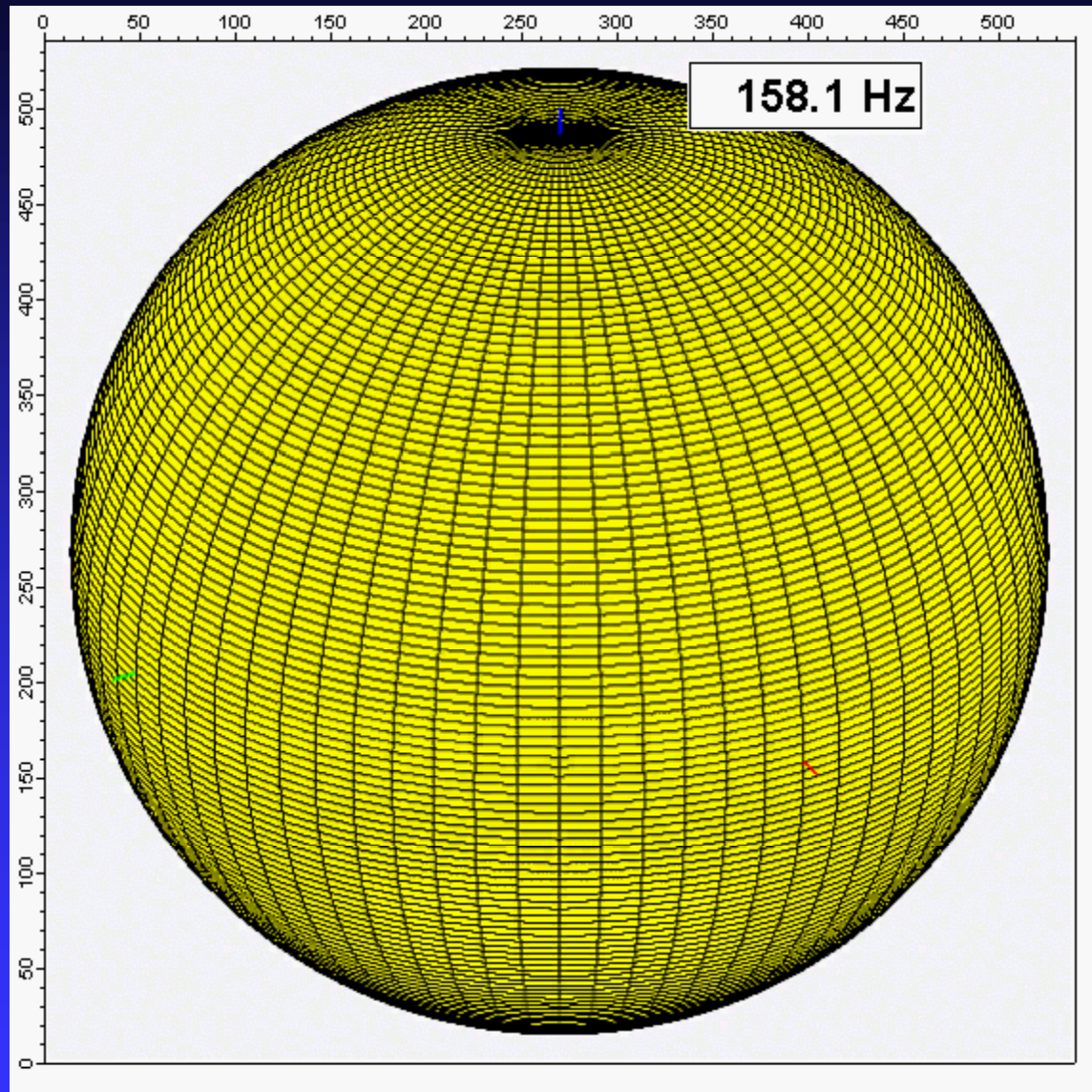
# CBT Curved Array: Broad Vertical Coverage Cont.



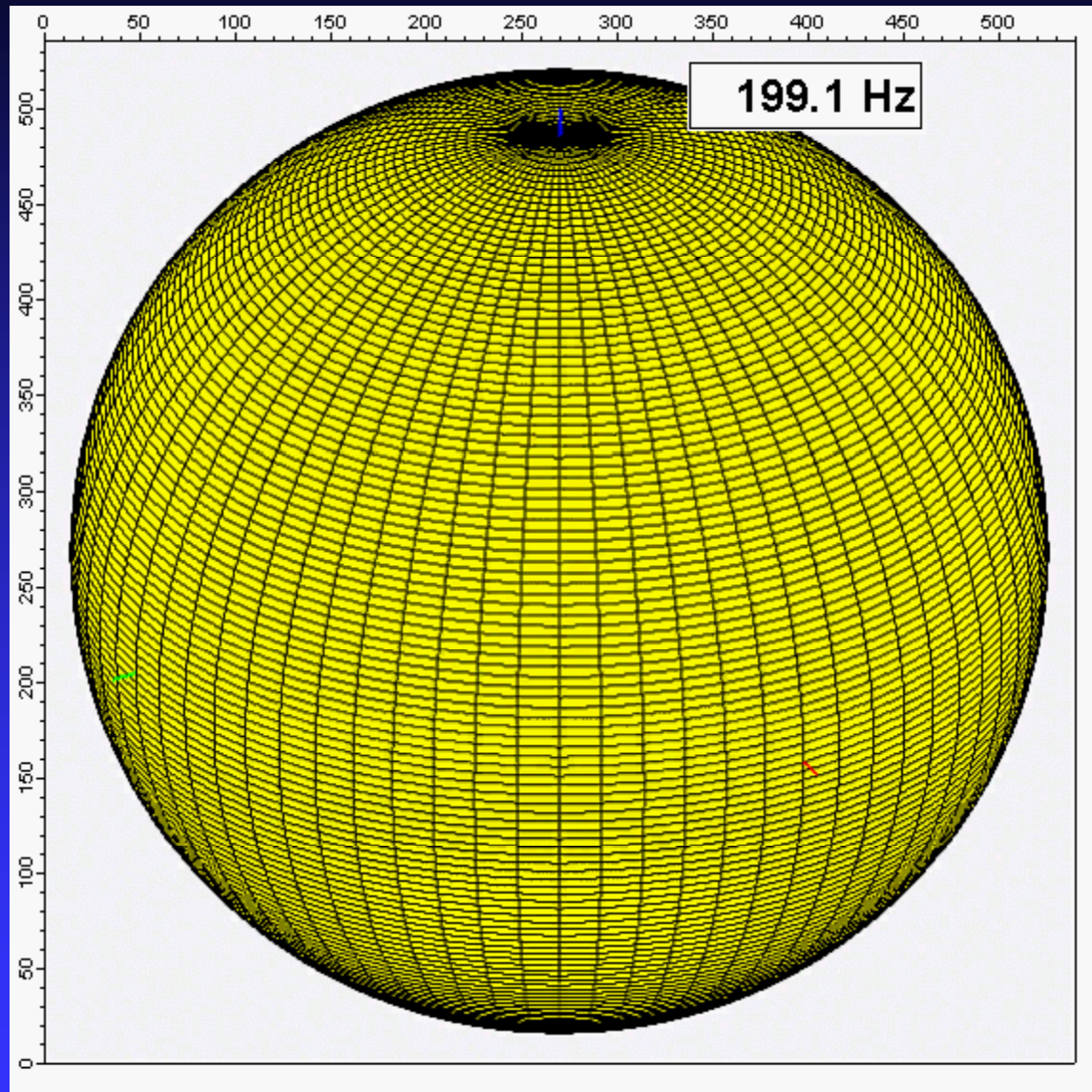
# CBT Curved Array: Broad Vertical Coverage Cont.



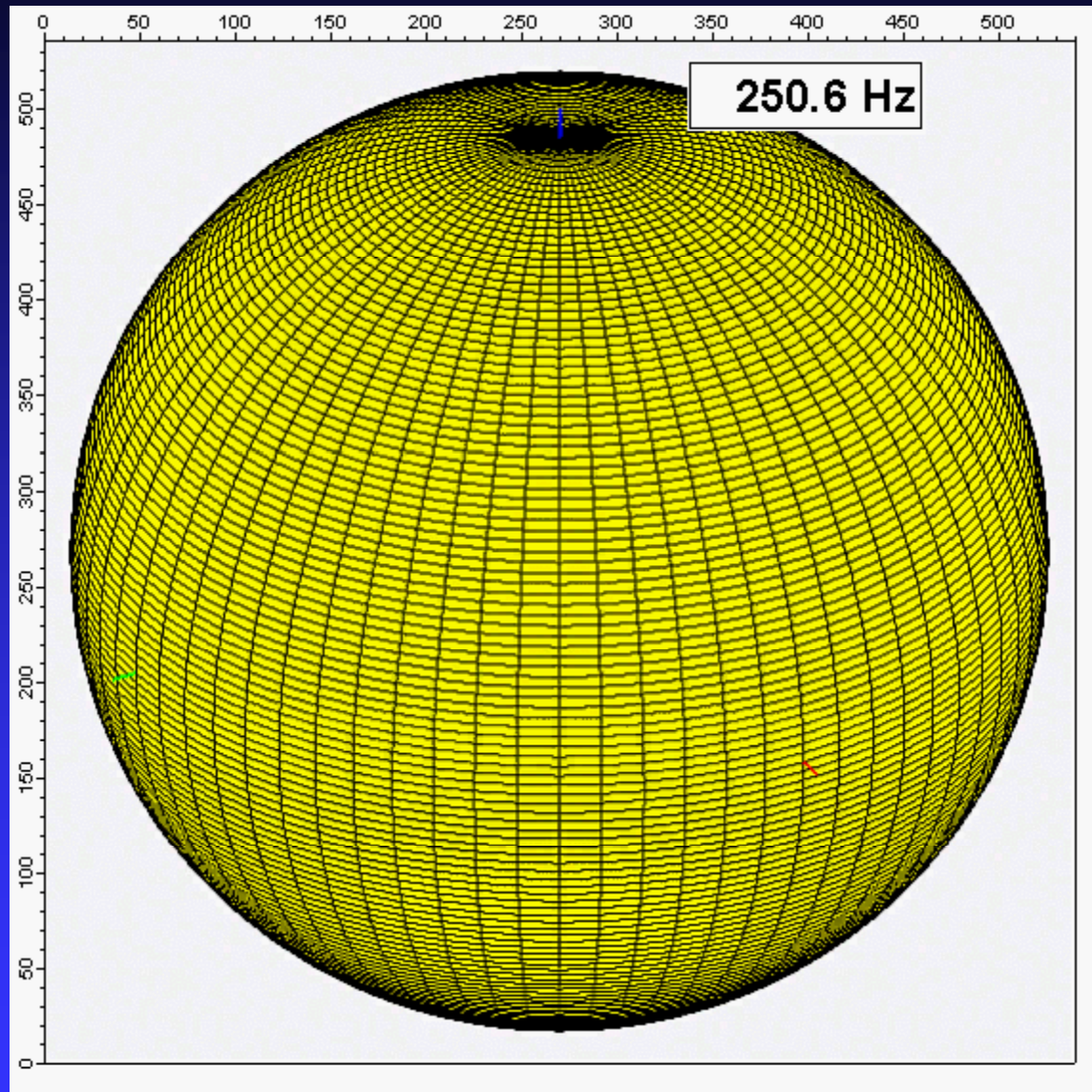
# CBT Curved Array: Broad Vertical Coverage Cont.



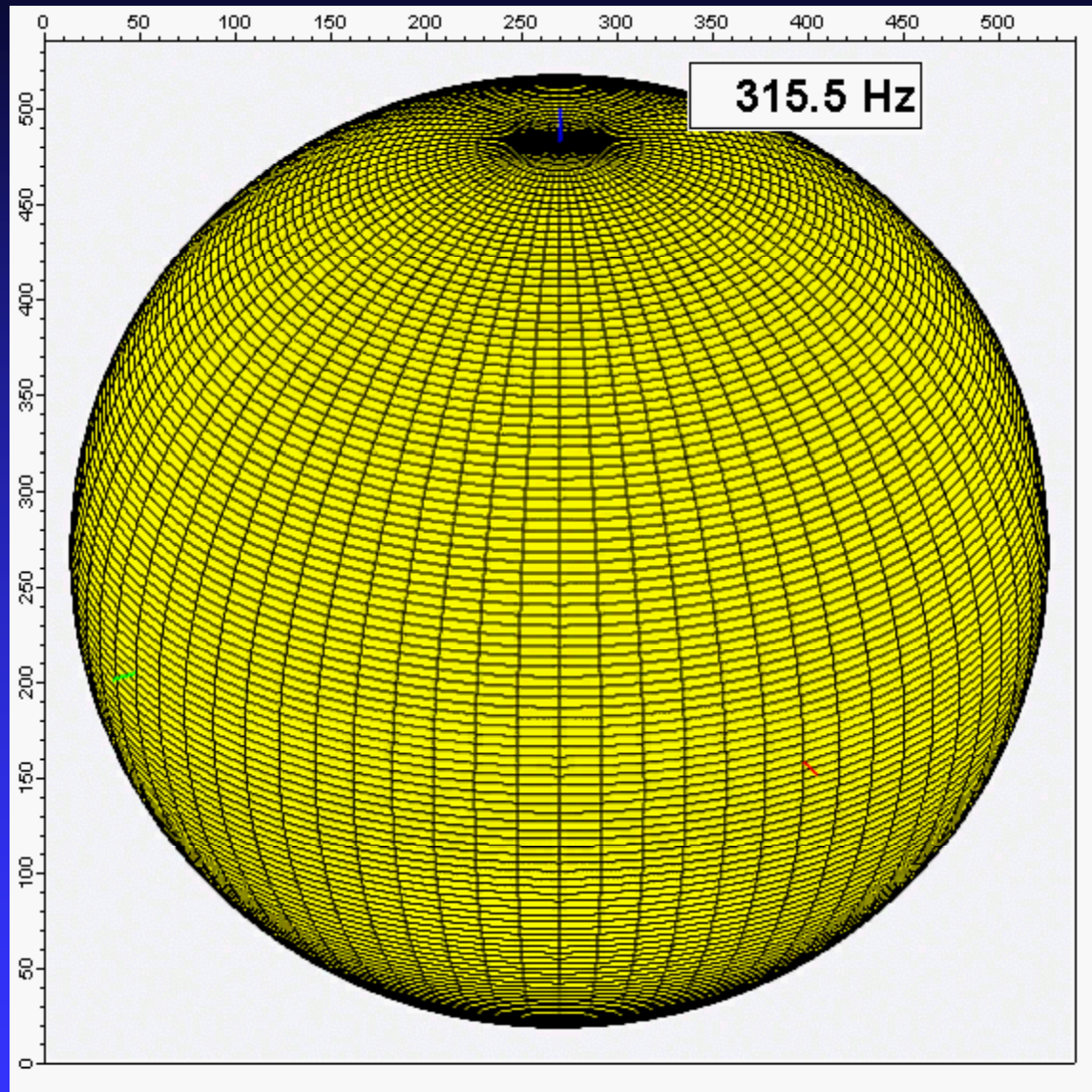
# CBT Curved Array: Broad Vertical Coverage Cont.



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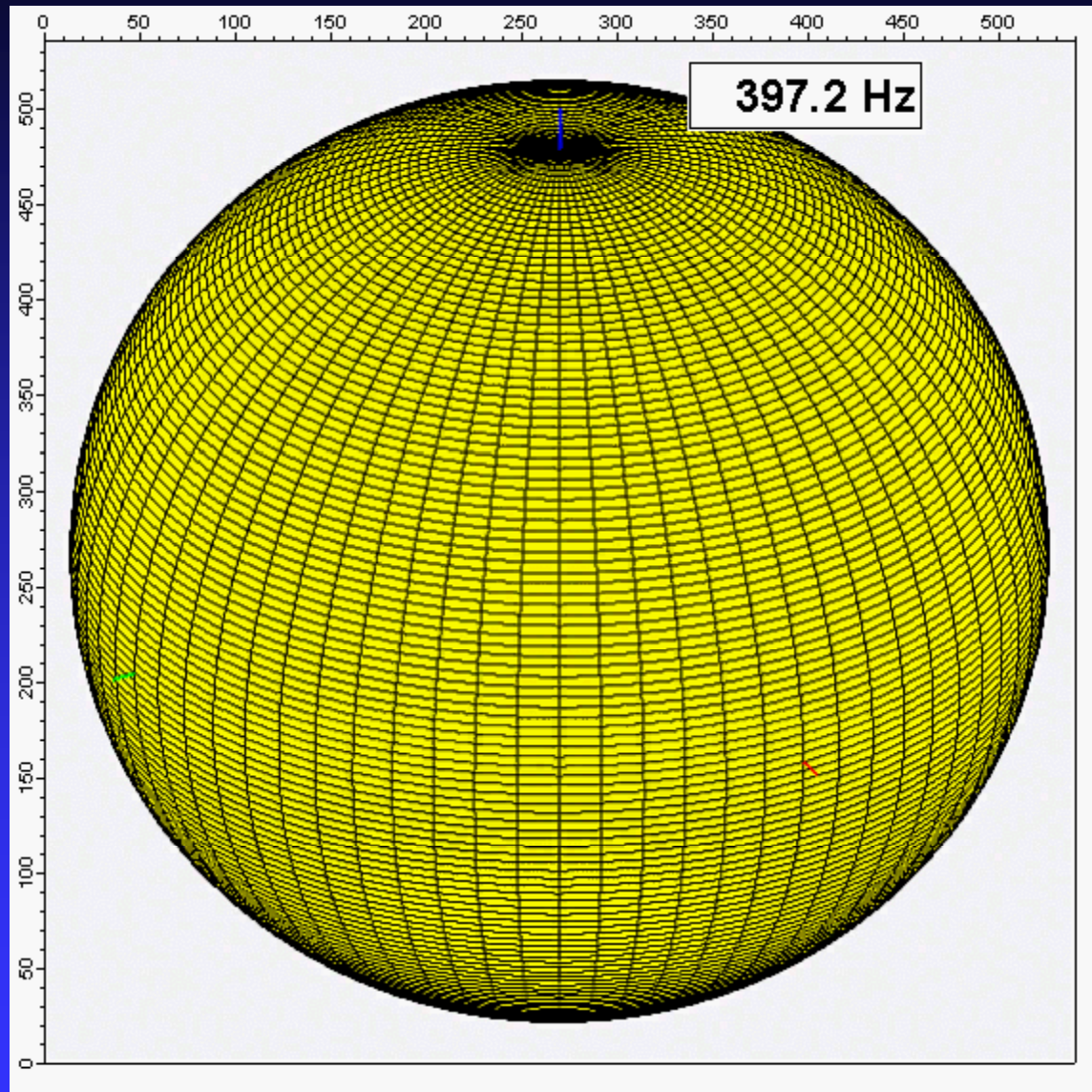


# CBT Curved Array: Broad Vertical Coverage Cont.

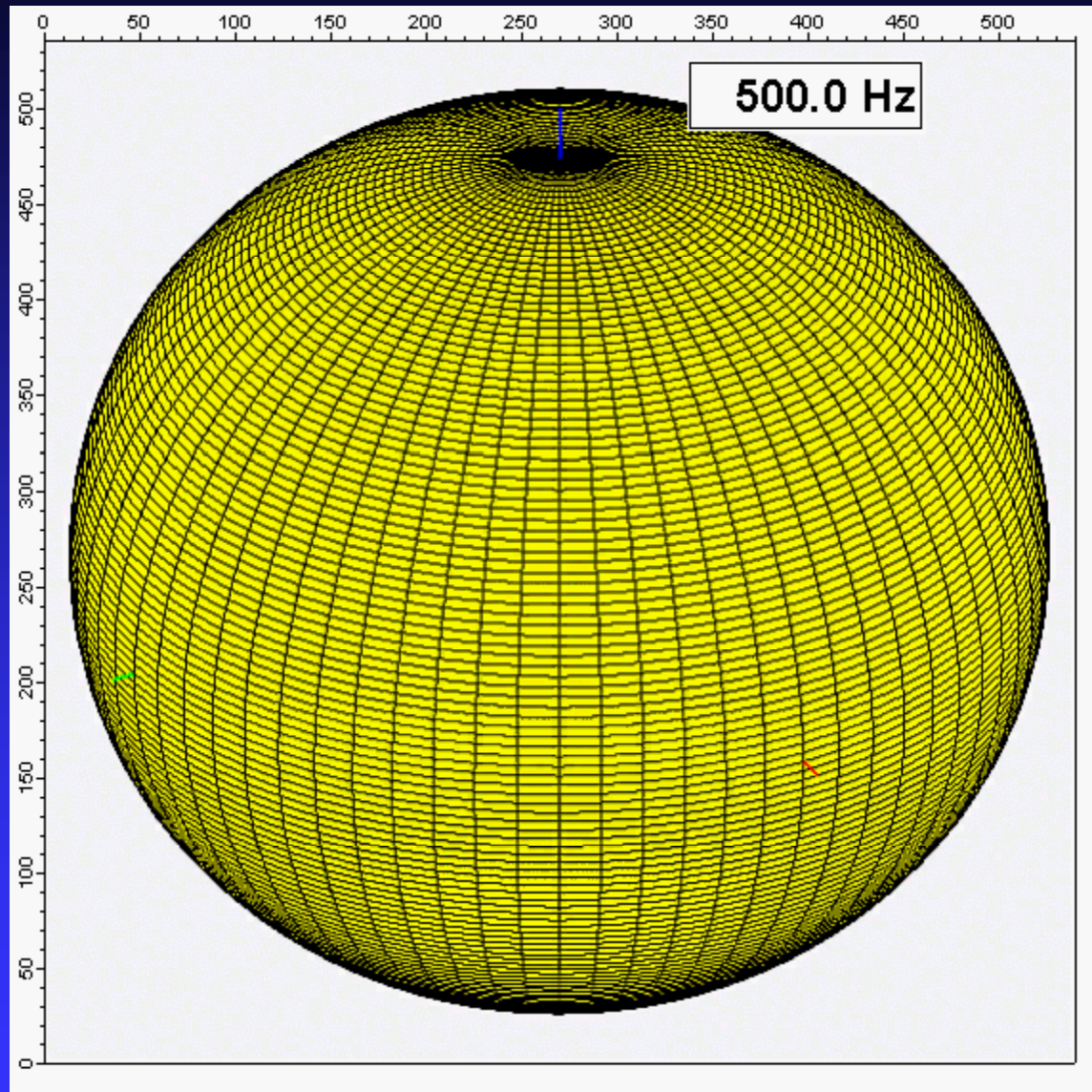




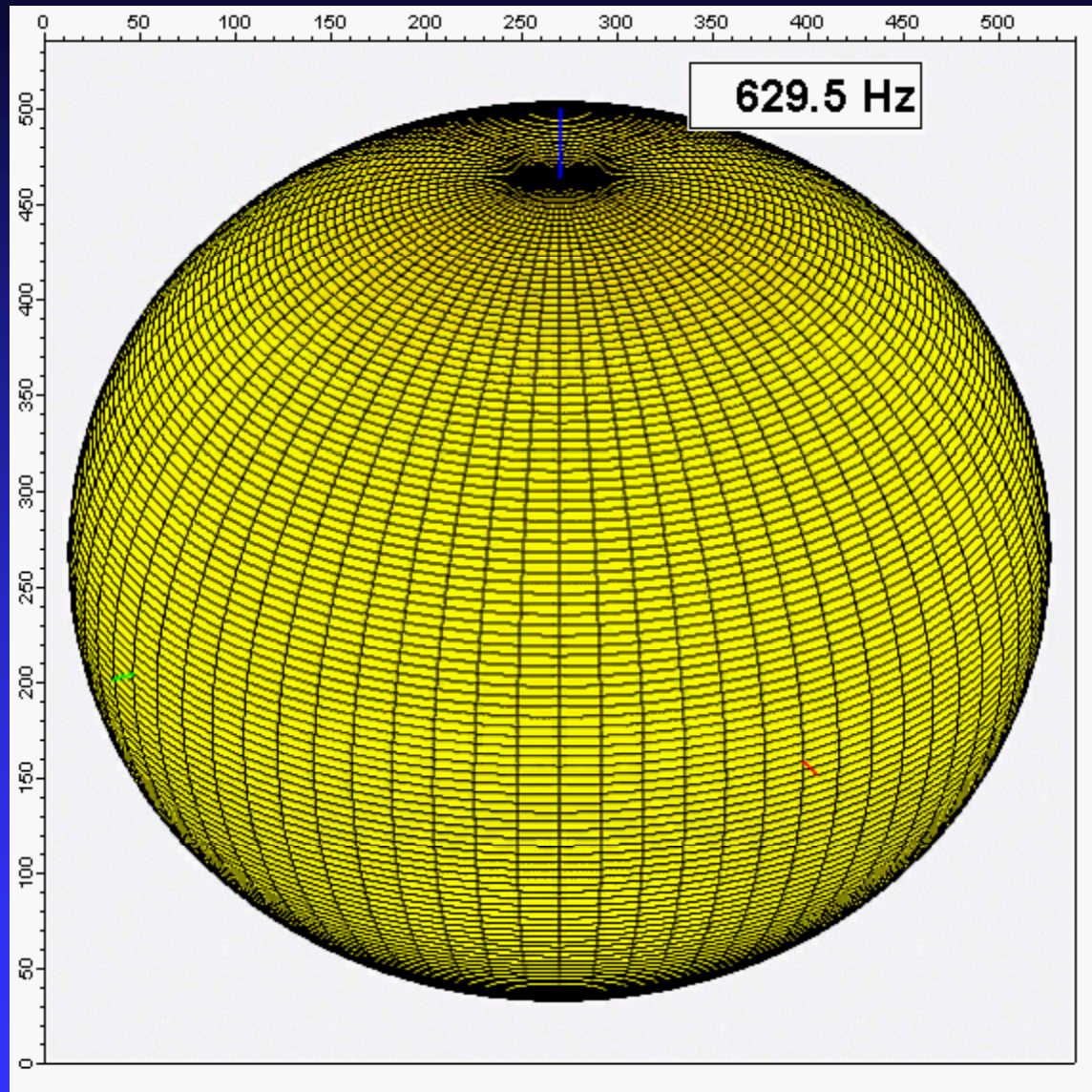
# CBT Curved Array: Broad Vertical Coverage Cont.



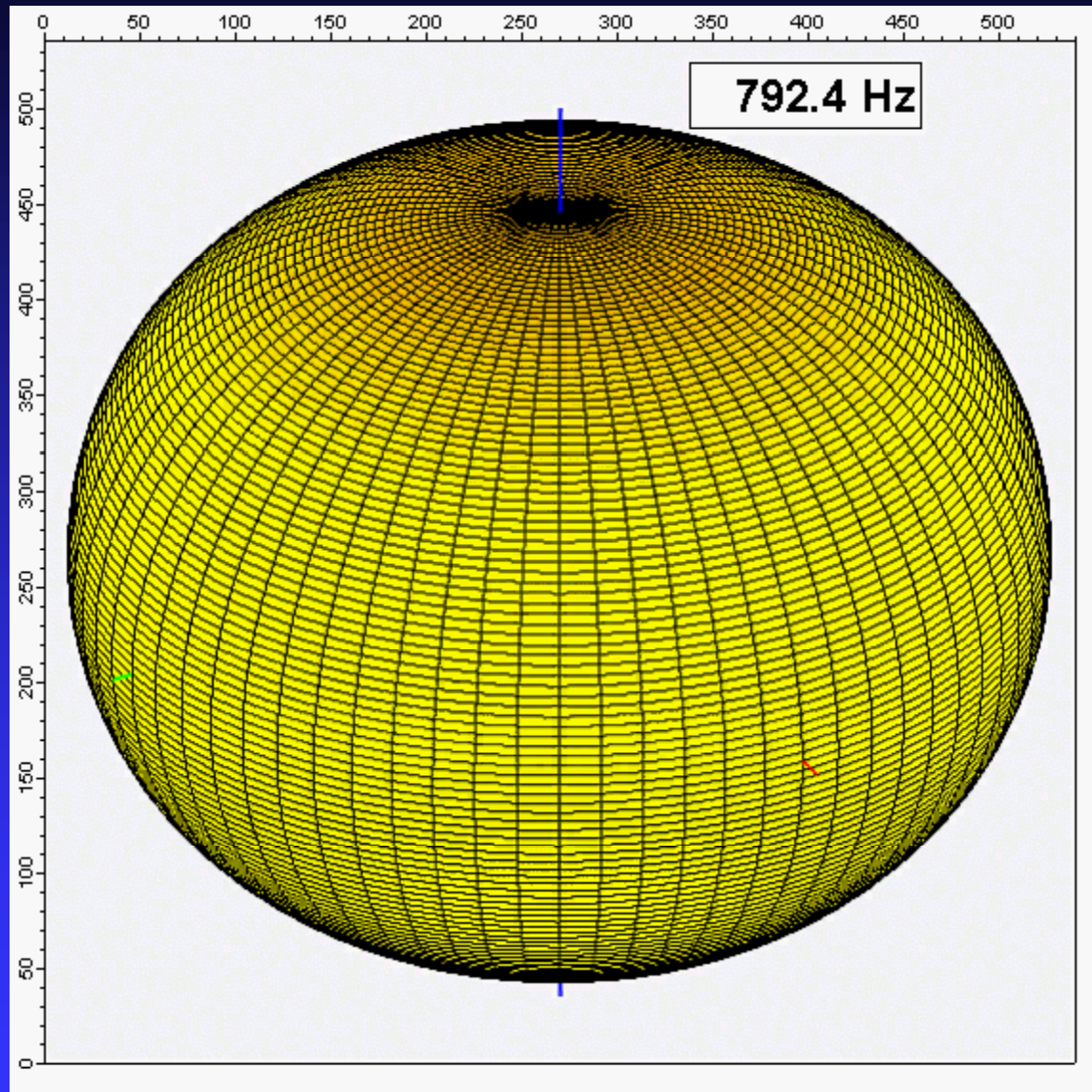
# CBT Curved Array: Broad Vertical Coverage Cont.



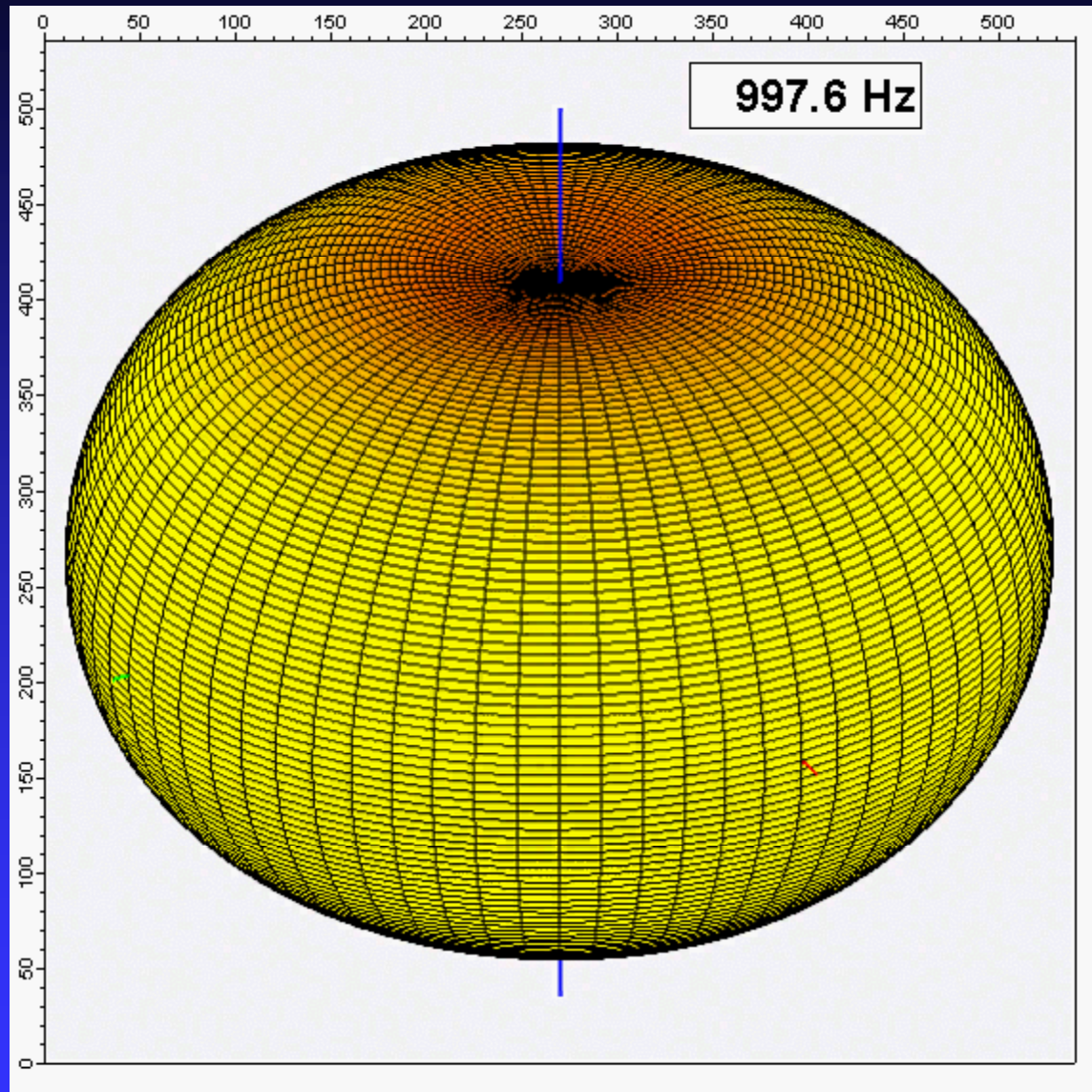
# CBT Curved Array: Broad Vertical Coverage Cont.



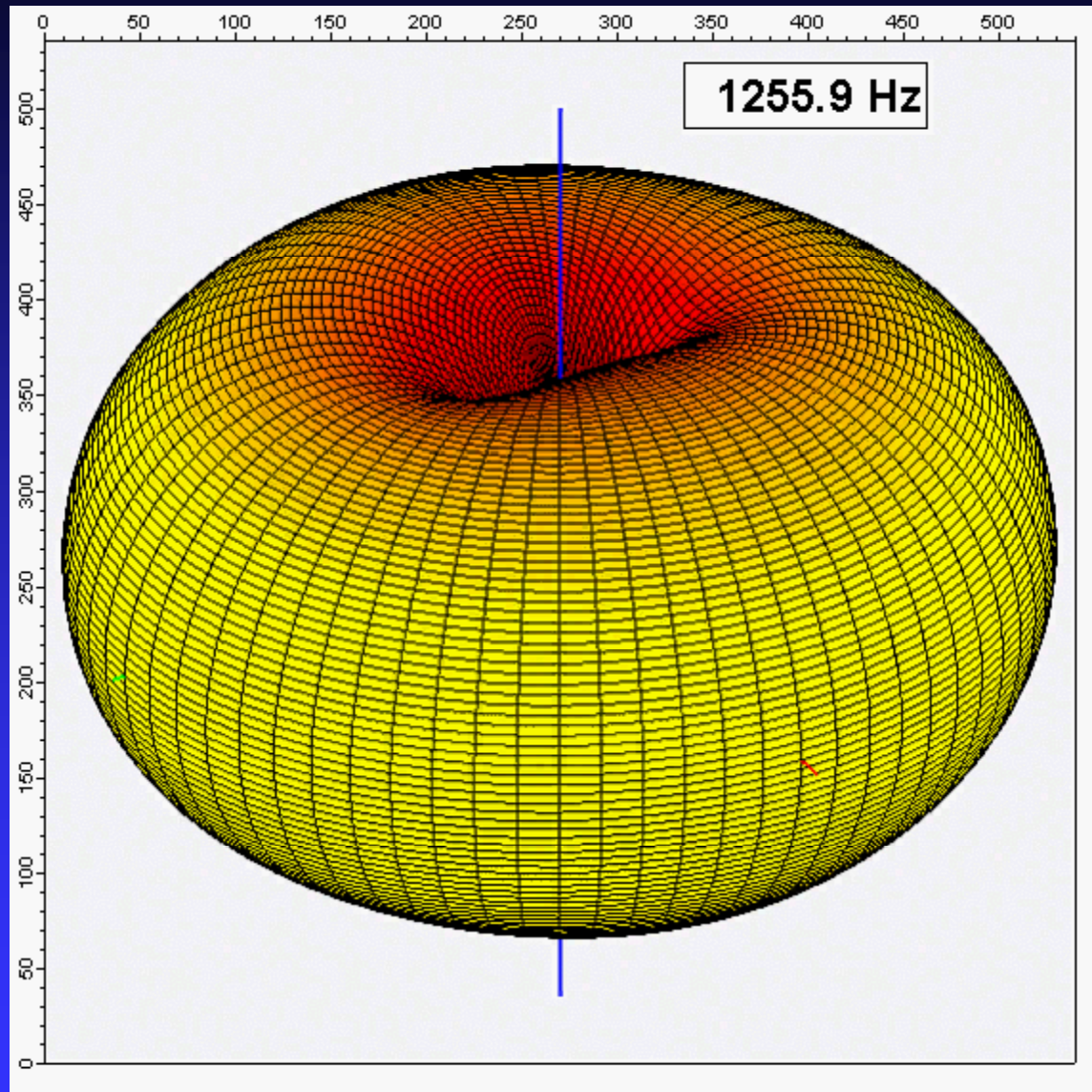
# CBT Curved Array: Broad Vertical Coverage Cont.



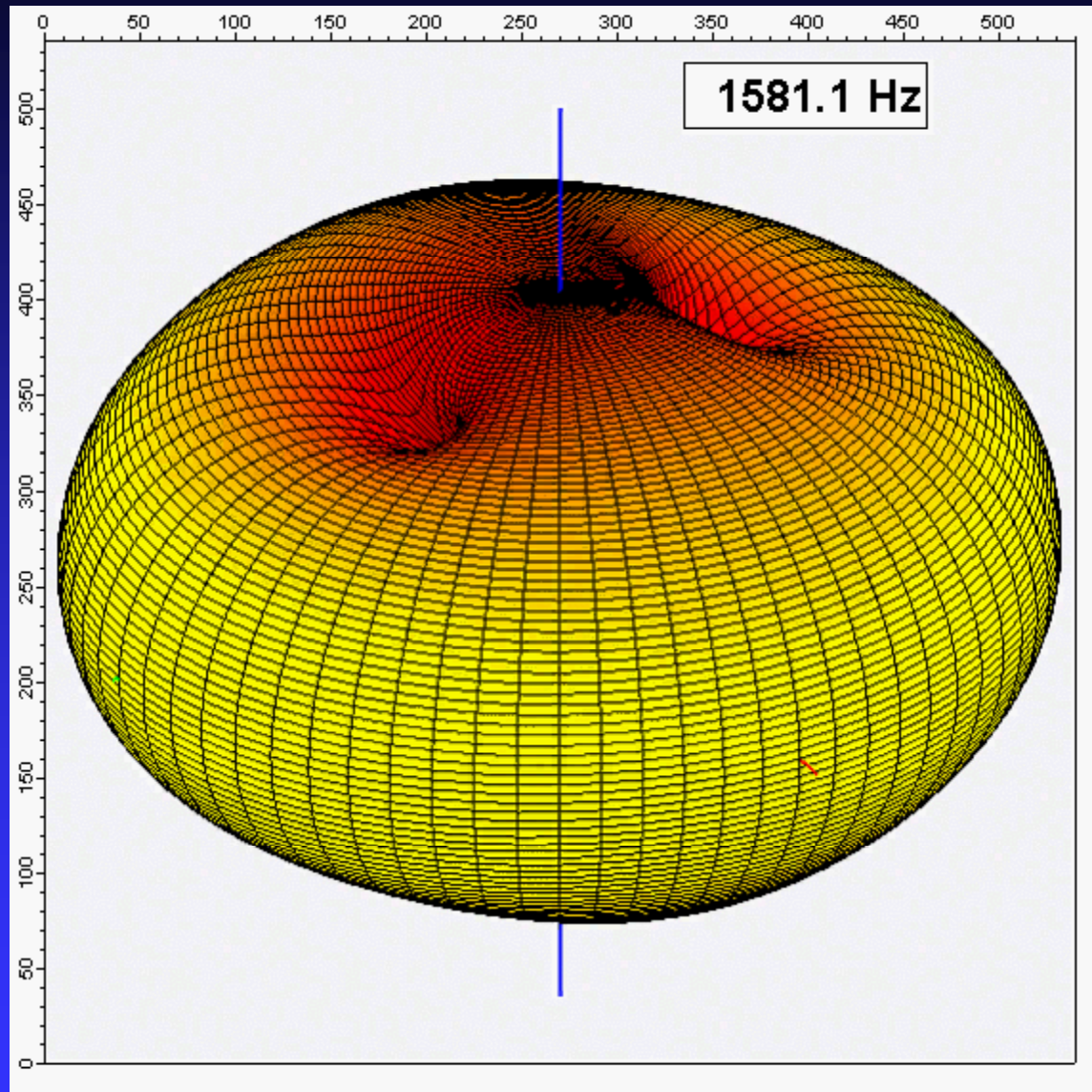
# CBT Curved Array: Broad Vertical Coverage Cont.



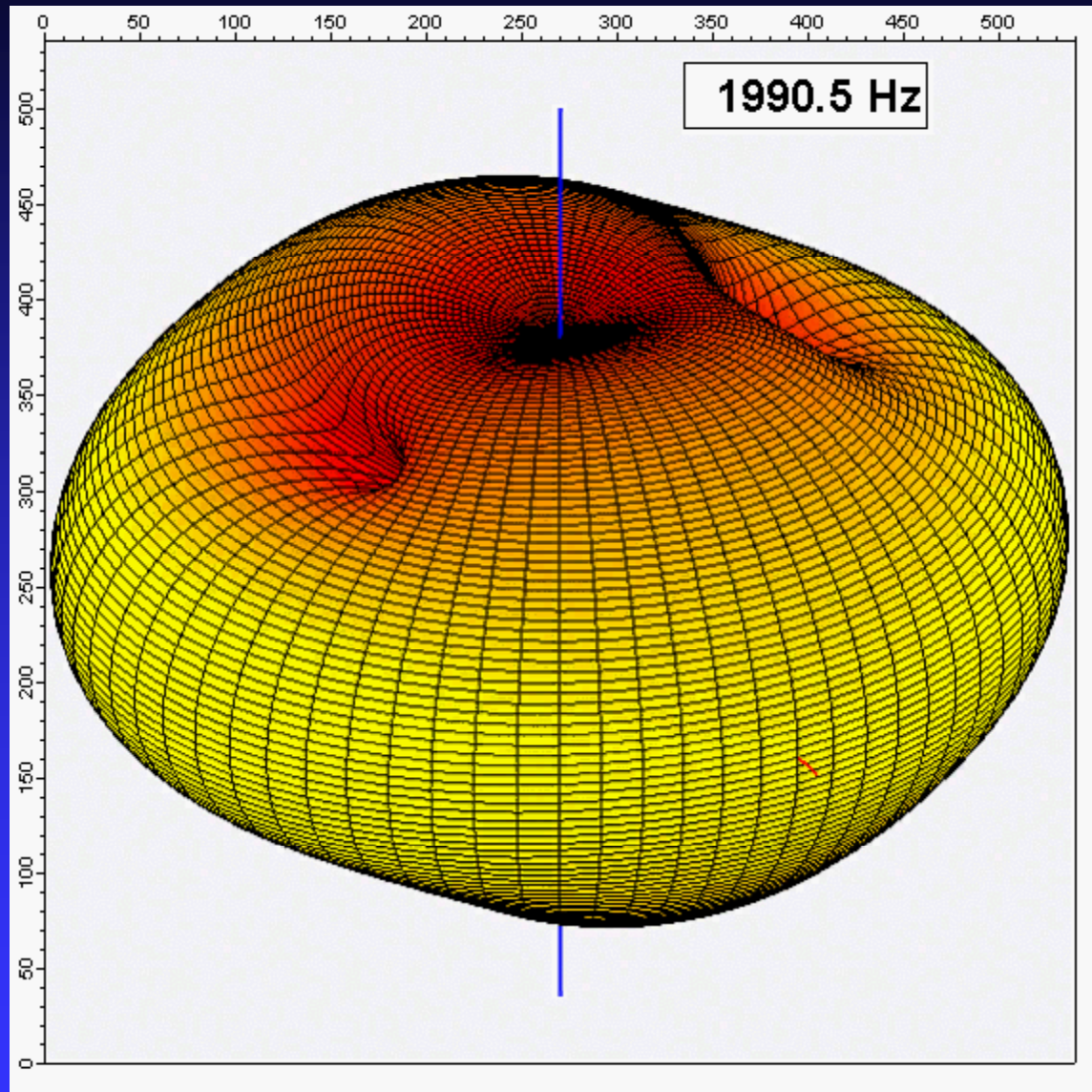
# CBT Curved Array: Broad Vertical Coverage Cont.



# CBT Curved Array: Broad Vertical Coverage Cont.

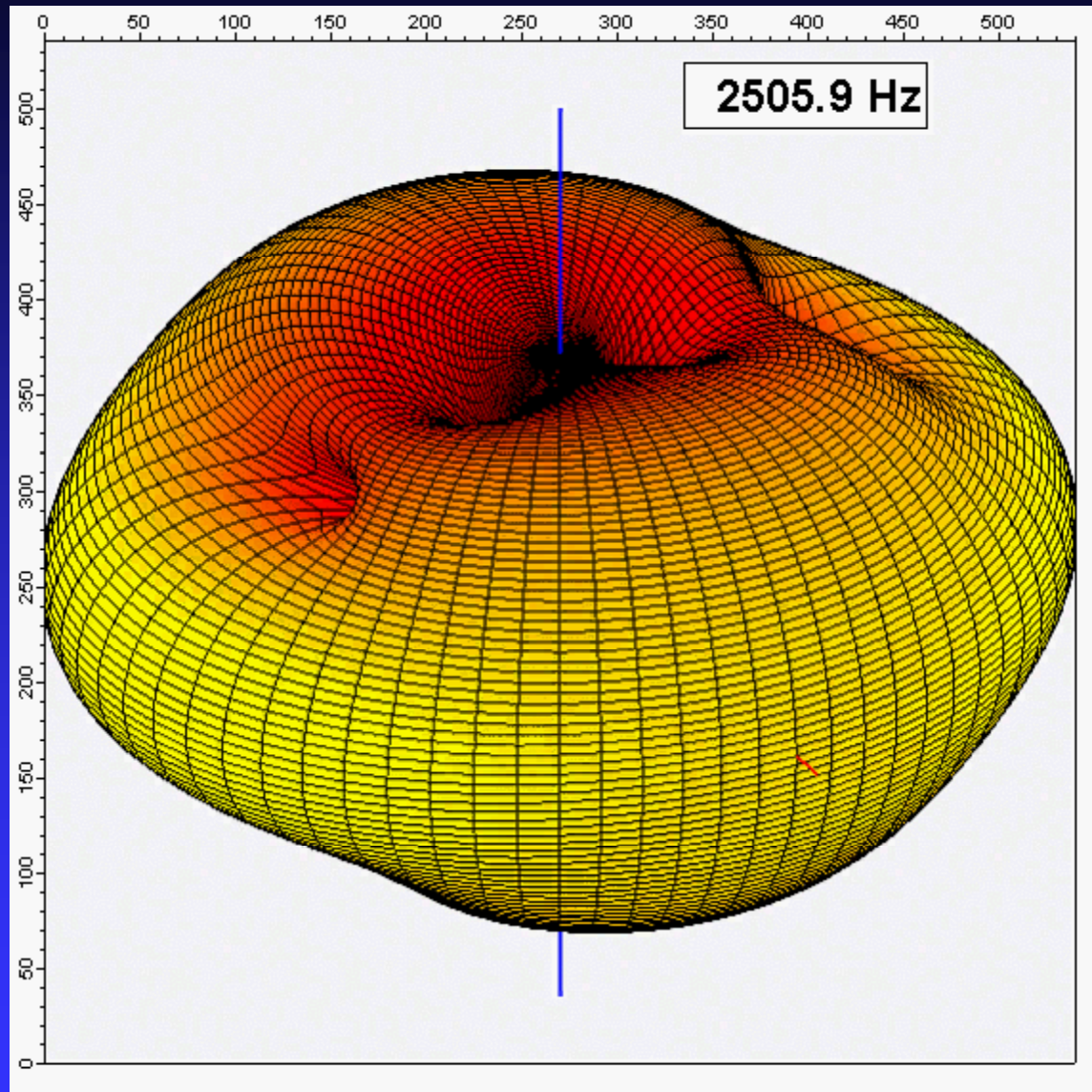


# CBT Curved Array: Broad Vertical Coverage Cont.

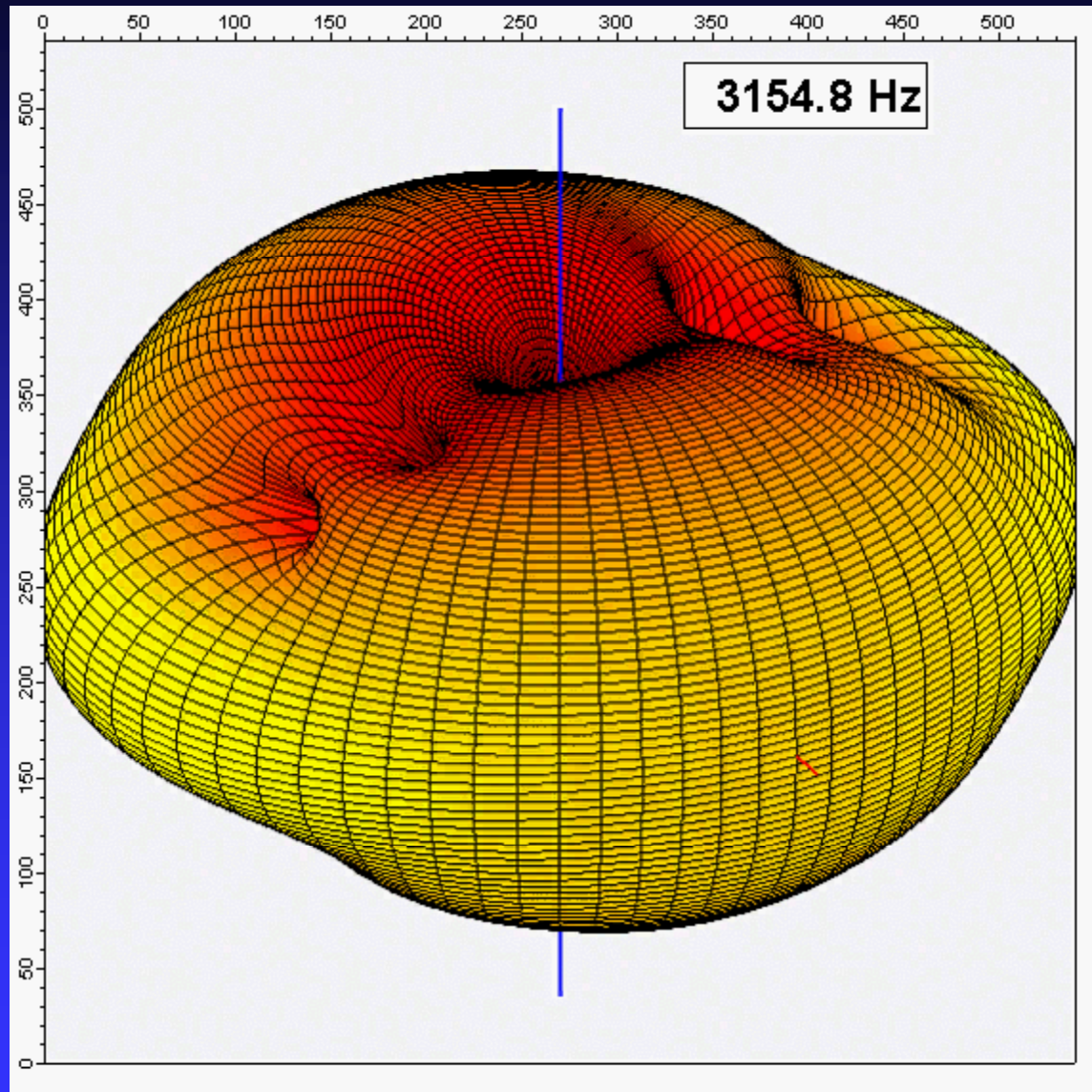




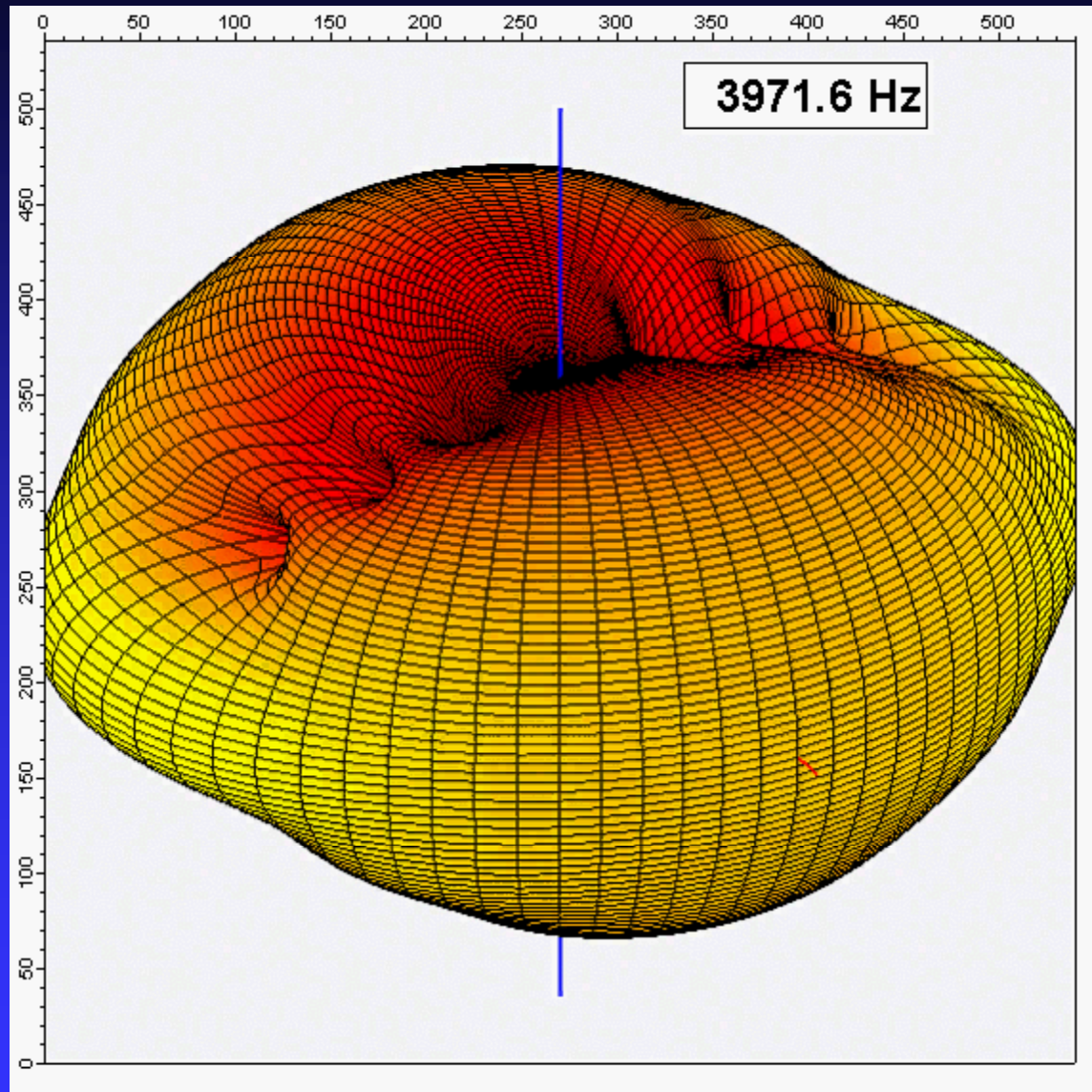
# CBT Curved Array: Broad Vertical Coverage Cont.



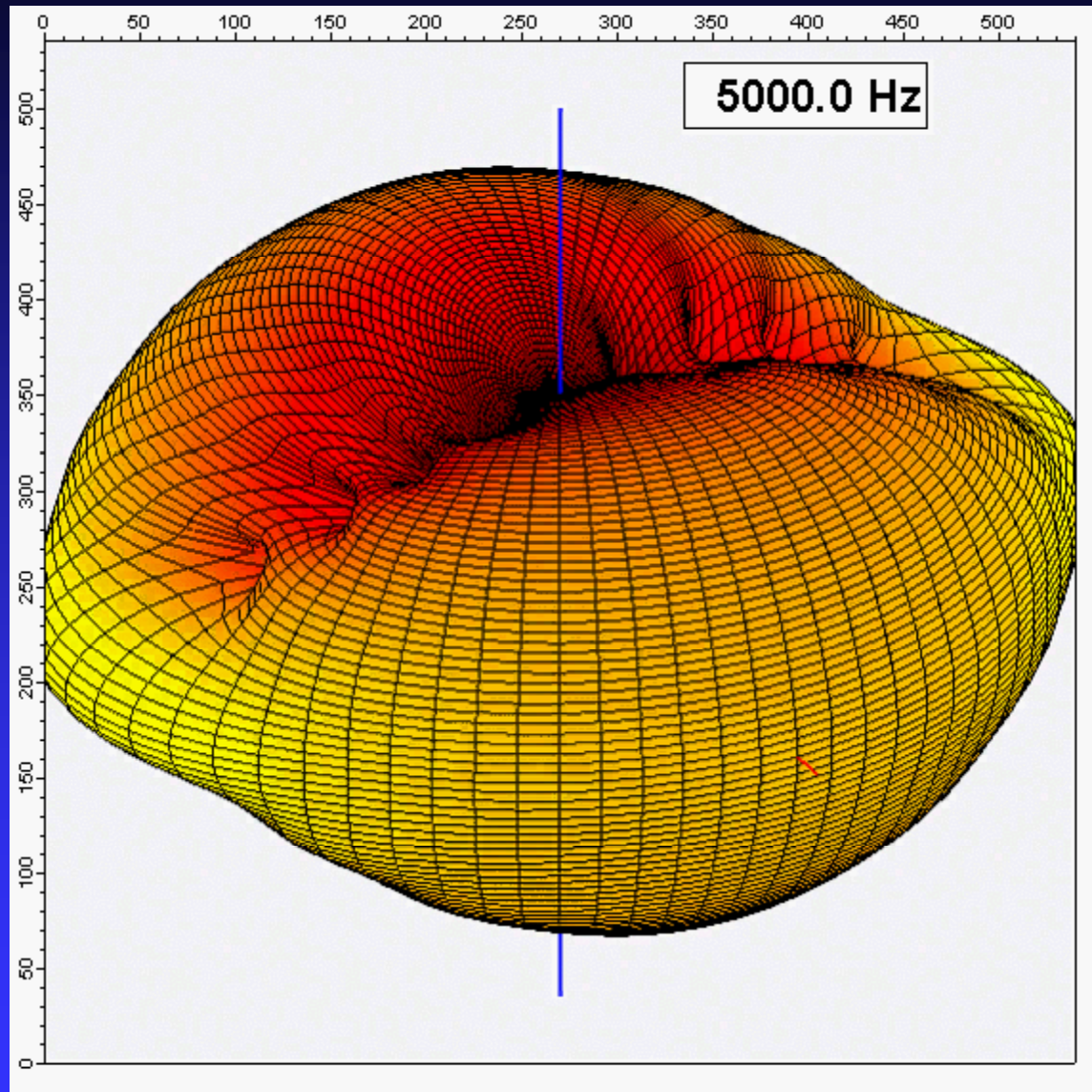
# CBT Curved Array: Broad Vertical Coverage Cont.



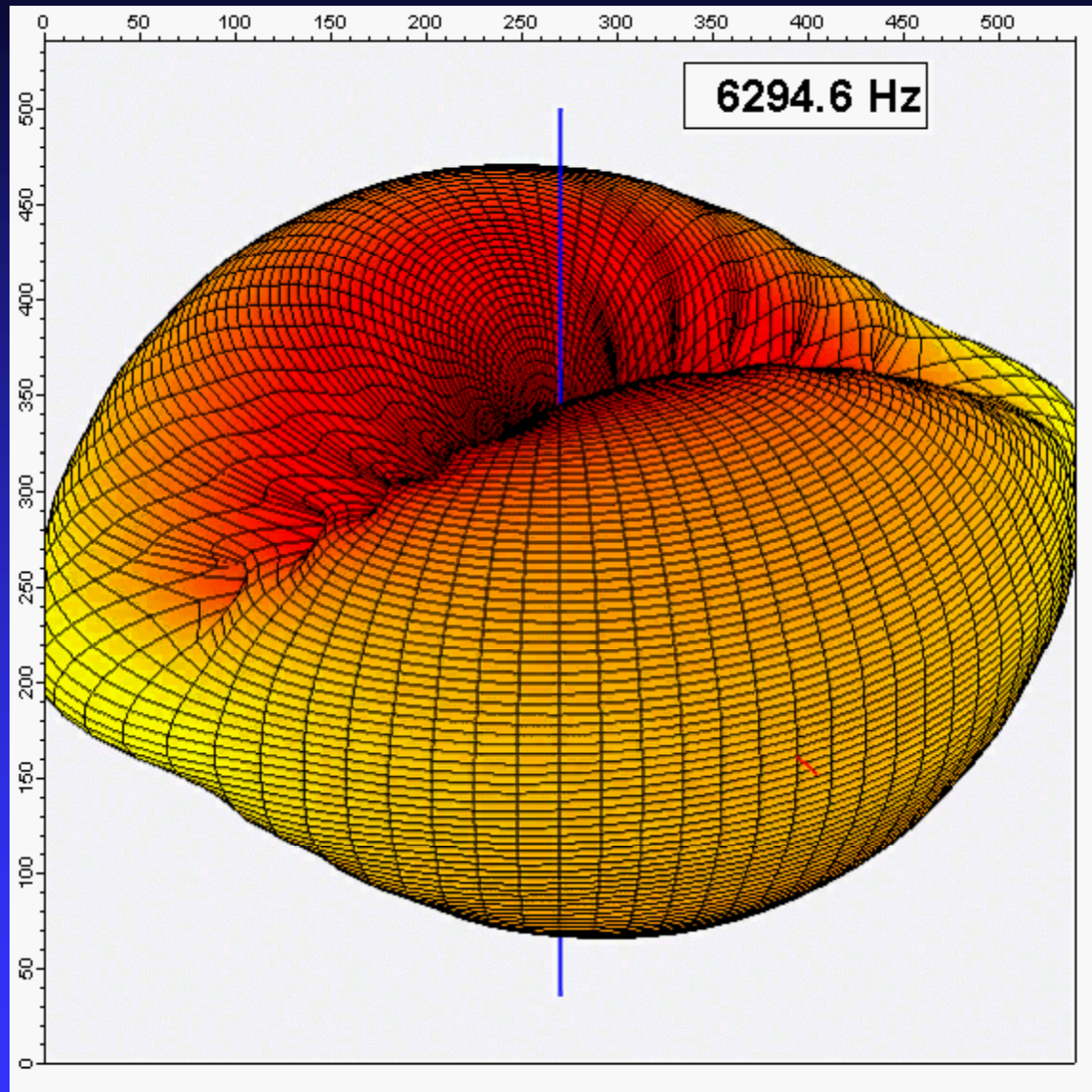
# CBT Curved Array: Broad Vertical Coverage Cont.



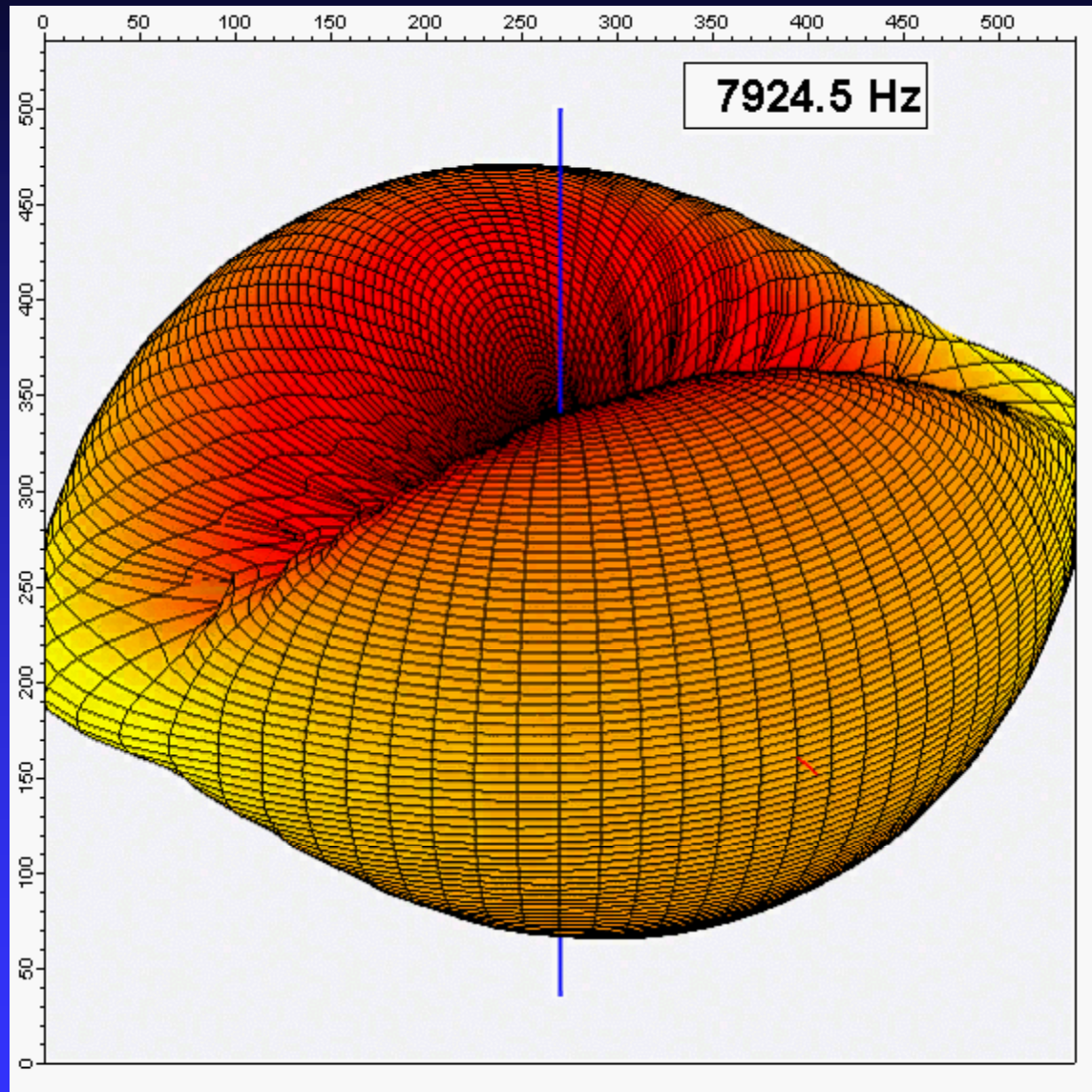
# CBT Curved Array: Broad Vertical Coverage Cont.



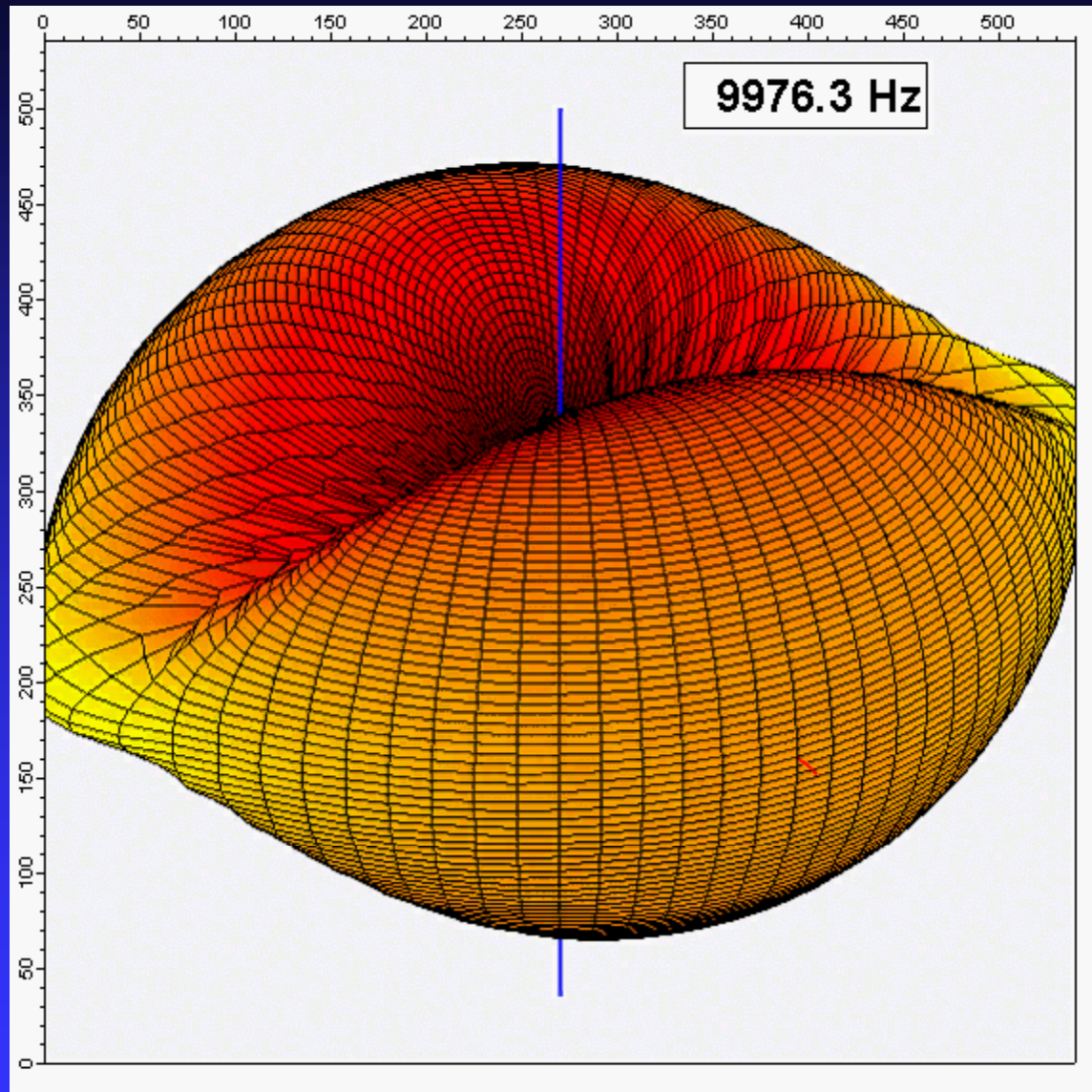
# CBT Curved Array: Broad Vertical Coverage Cont.



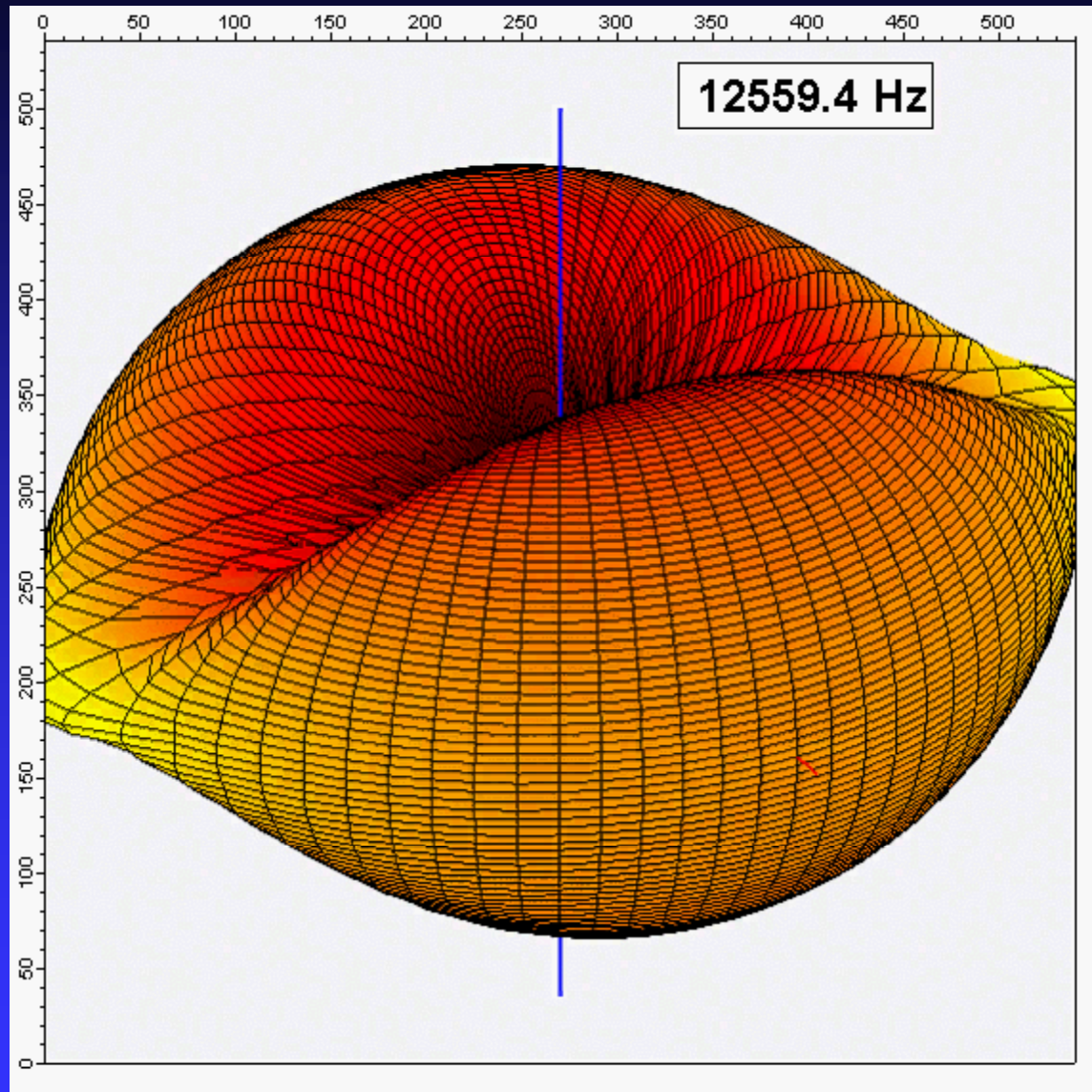
# CBT Curved Array: Broad Vertical Coverage Cont.



# CBT Curved Array: Broad Vertical Coverage Cont.

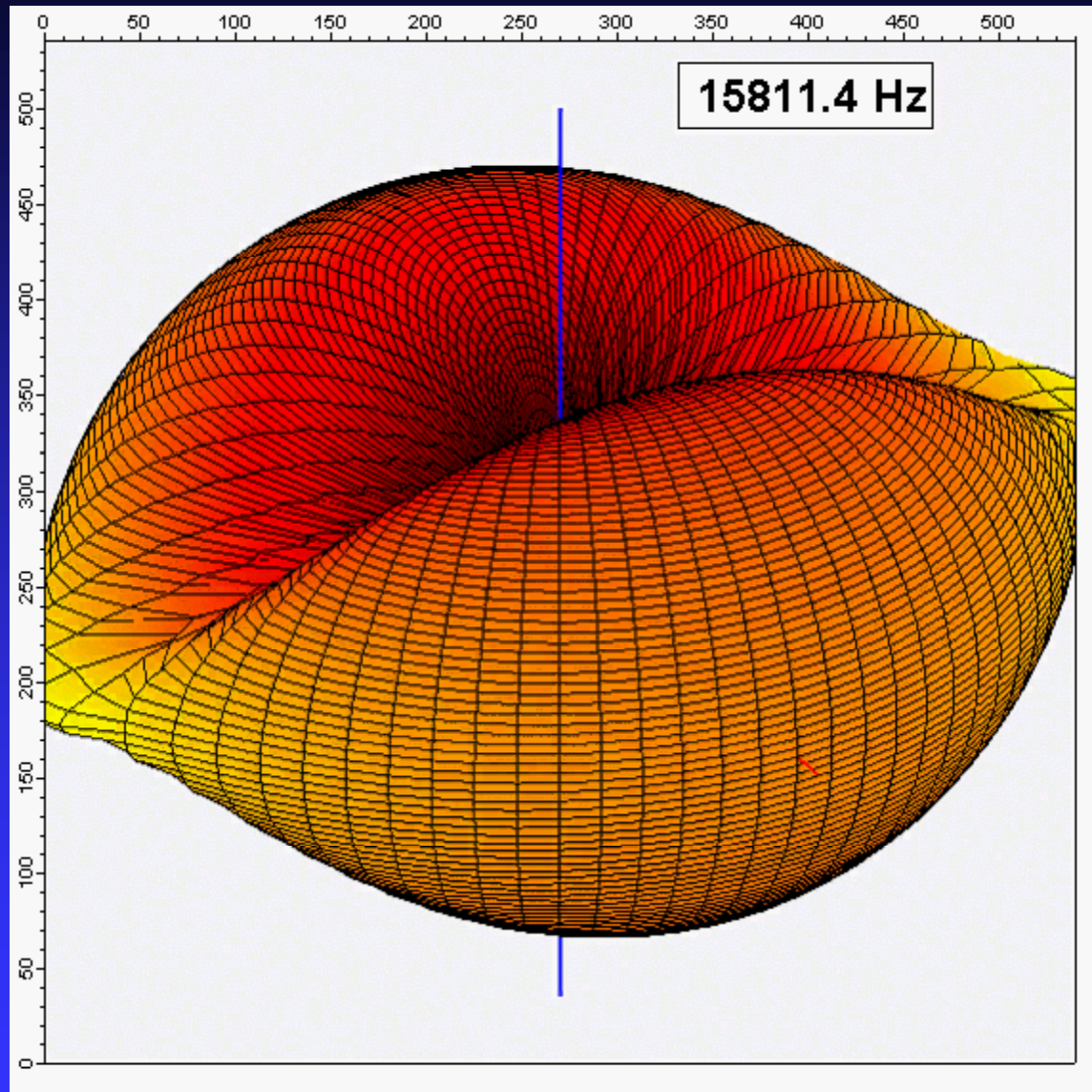


# CBT Curved Array: Broad Vertical Coverage Cont.





# CBT Curved Array: Broad Vertical Coverage Cont.



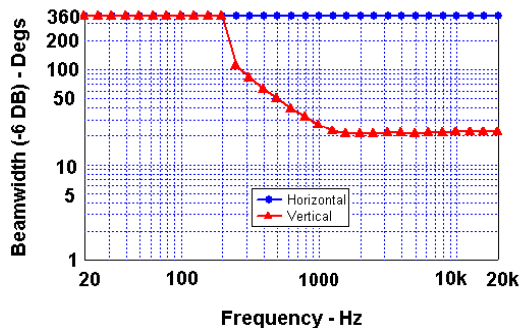
# CBT Curved Array: Broad Vertical Coverage Cont.

## End of CBT Broad Vertical Coverage Movie

# CBT Curved-Line Array Sound Radiation: Cont. Narrow Vertical Coverage

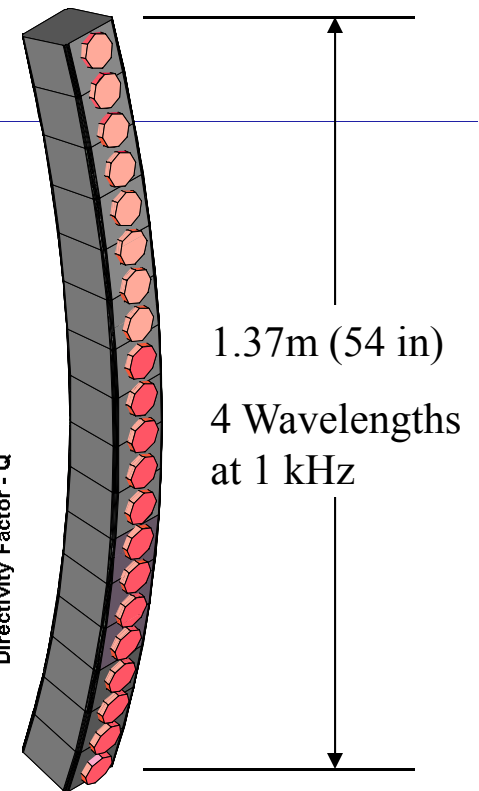
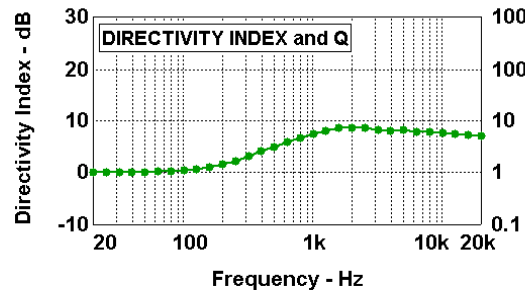
Assume a 200-point source CBT 35° curved-line array with Legendre amplitude shading providing a 22.5° vertical coverage above 1 kHz.

Beamwidth vs. Frequency



**Is 4 Times Higher!**

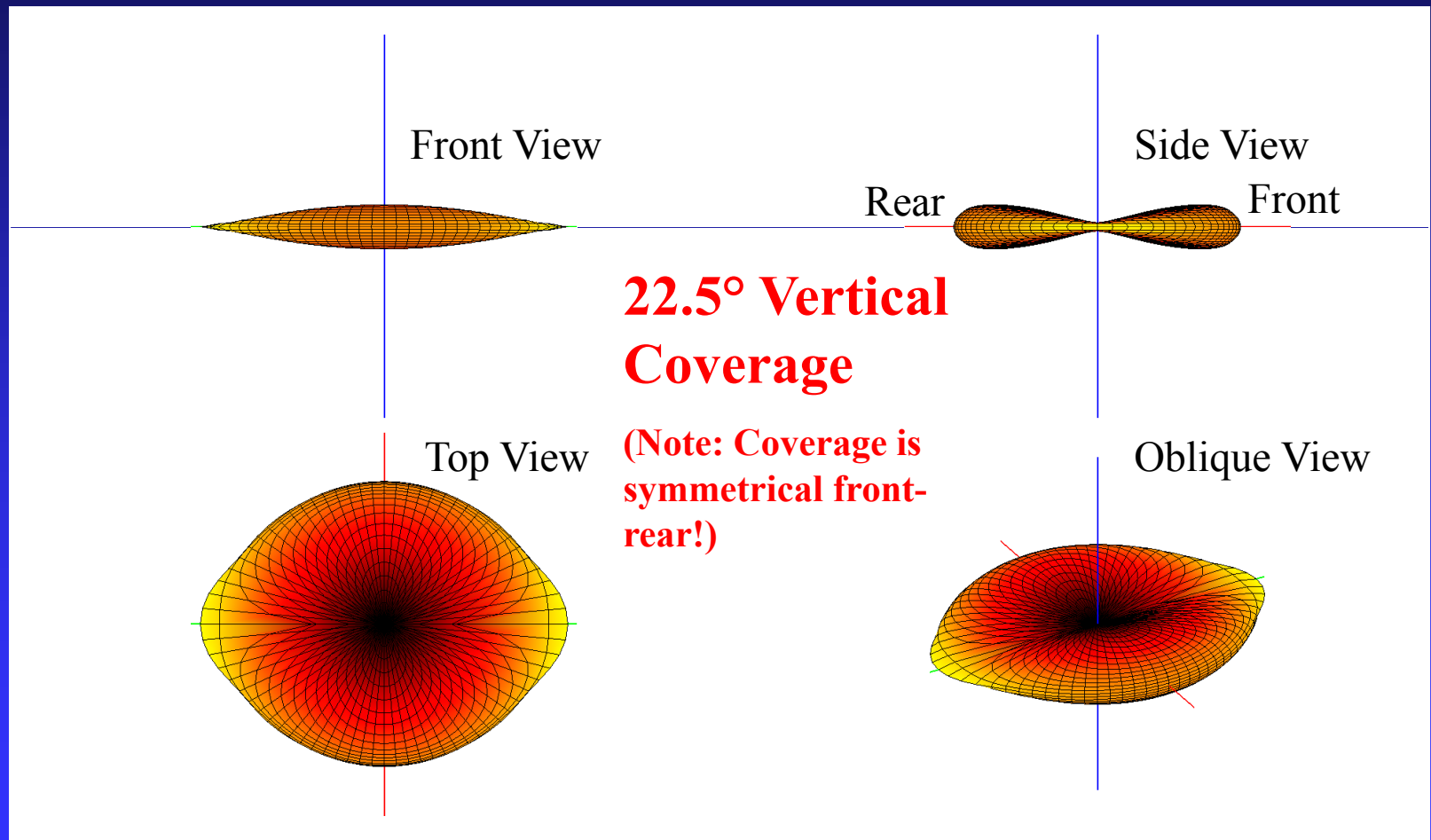
Directivity vs. Frequency



# CBT Curved-Line Array : Cont.

## Narrow Vertical Coverage

Balloon Plots at 8 kHz (Vertical Beamwidth = 22.5°)



# CBT Curved Array: Narrow Vertical Coverage Cont.

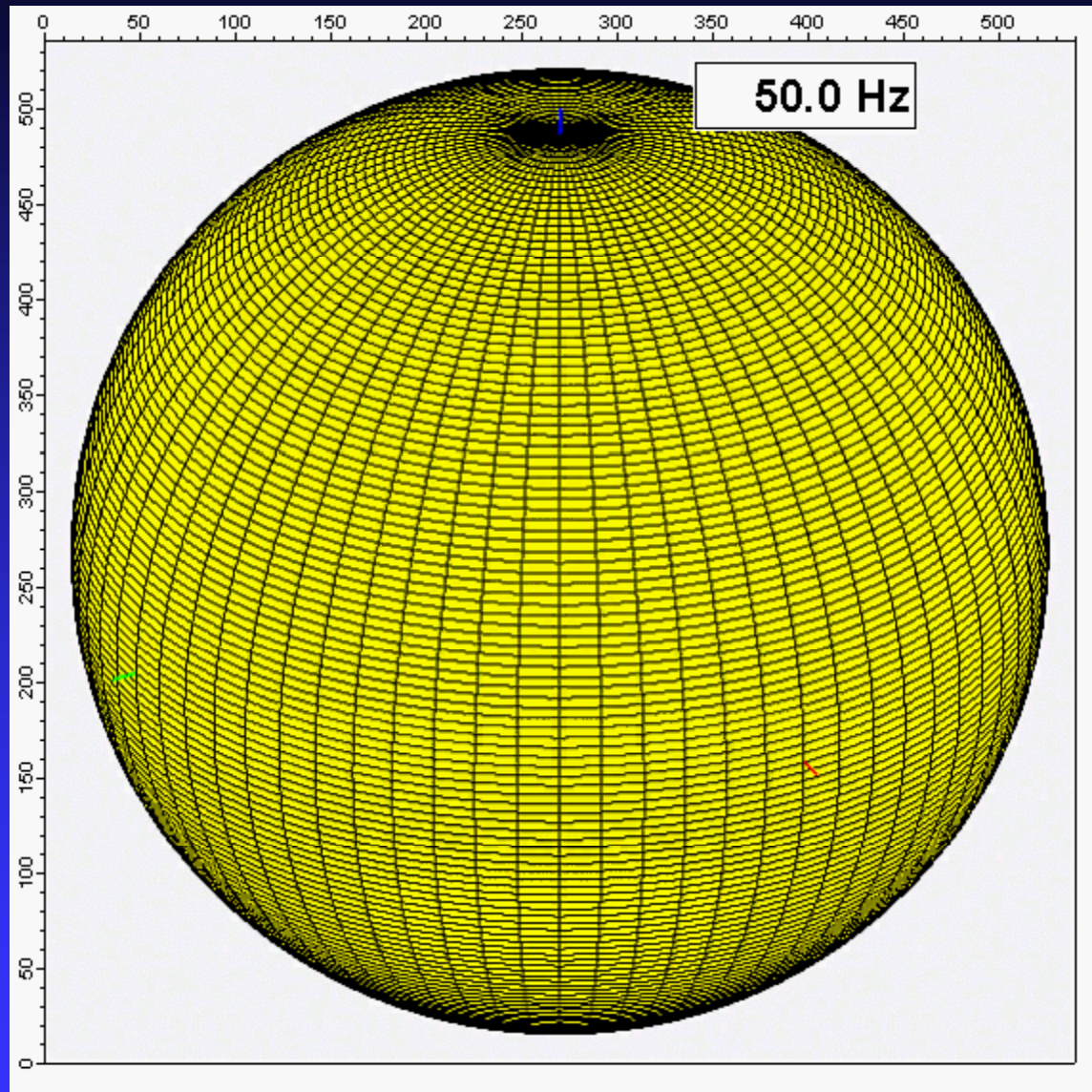
Play Quick Time Movie of 3D  
Radiation of Narrow Vertical  
Coverage CBT Line Array

(Oblique View Normalized to On Axis: : 50 Hz  
to 16 kHz at one-third-octave intervals)

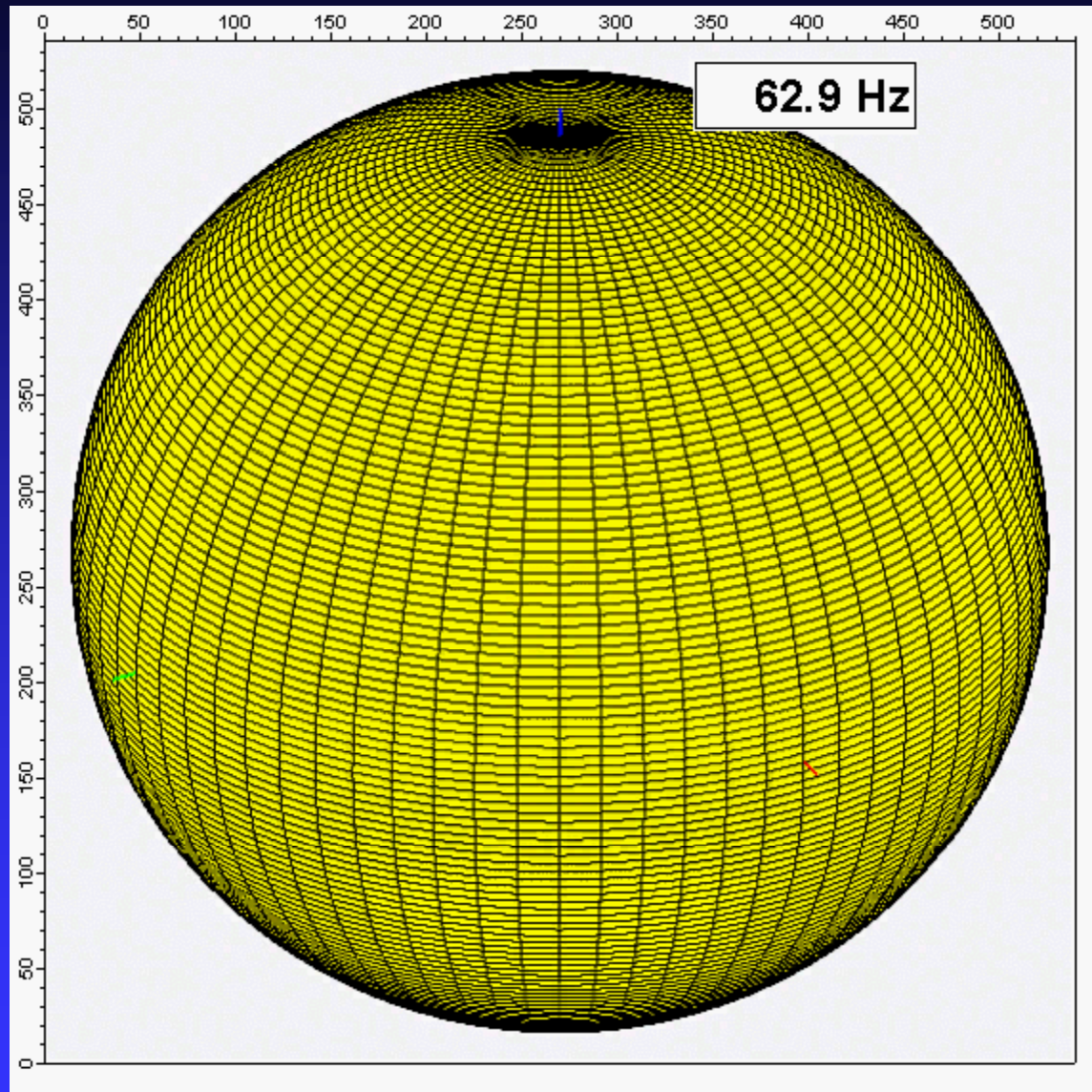
## Start of Movie

(Slides 78-103)

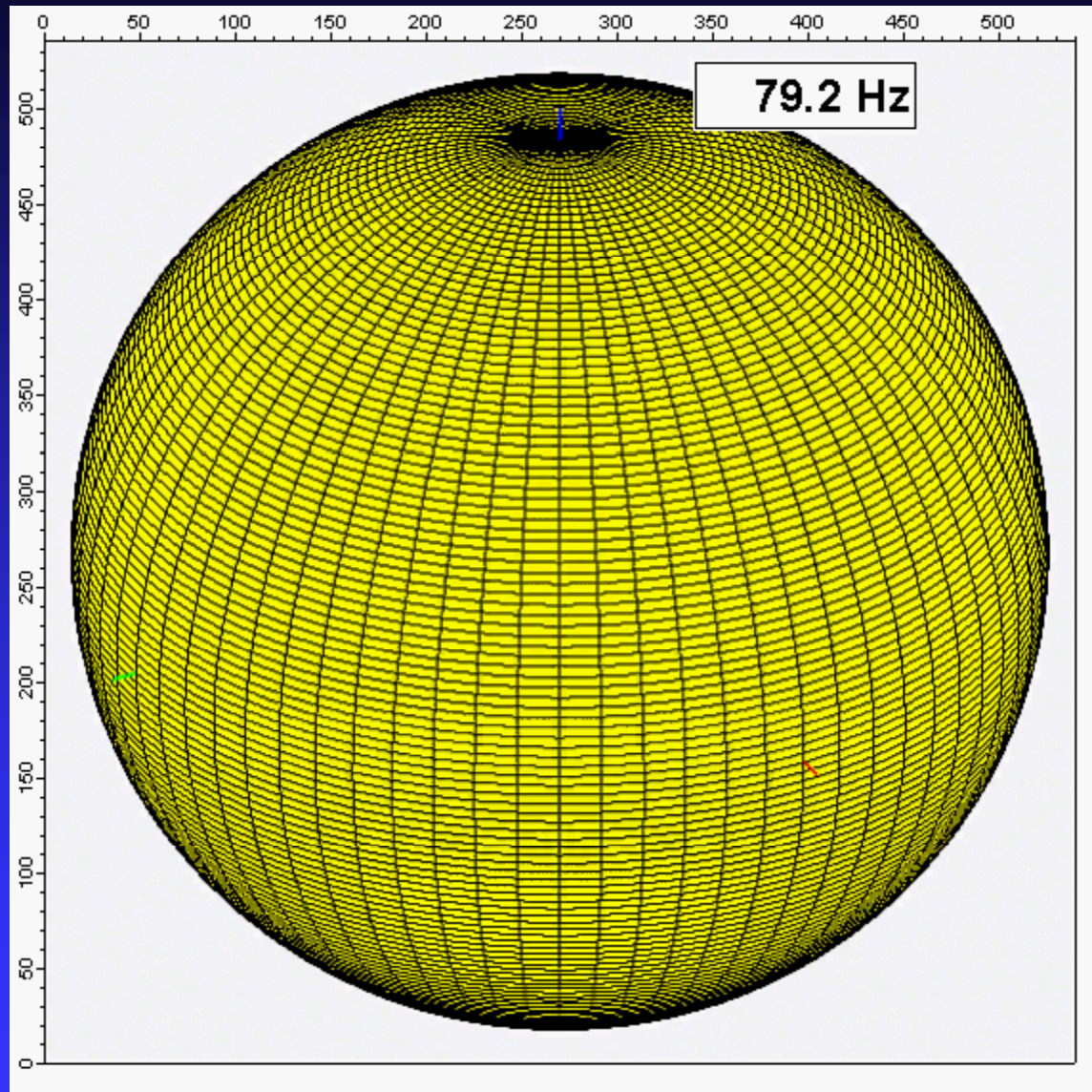
# CBT Curved Array: Narrow Vertical Coverage Cont.



# CBT Curved Array: Narrow Vertical Coverage Cont.

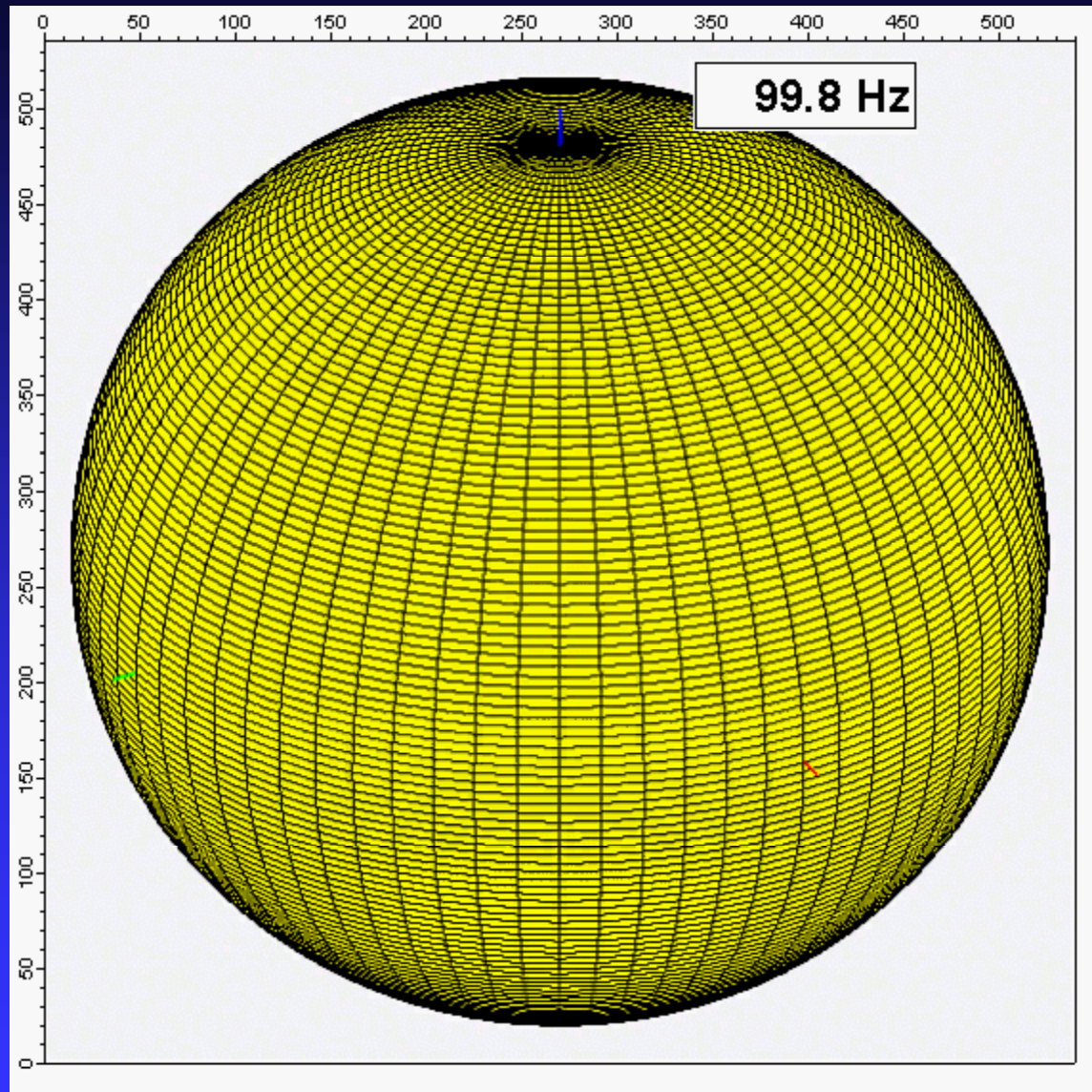


# CBT Curved Array: Narrow Vertical Coverage Cont.

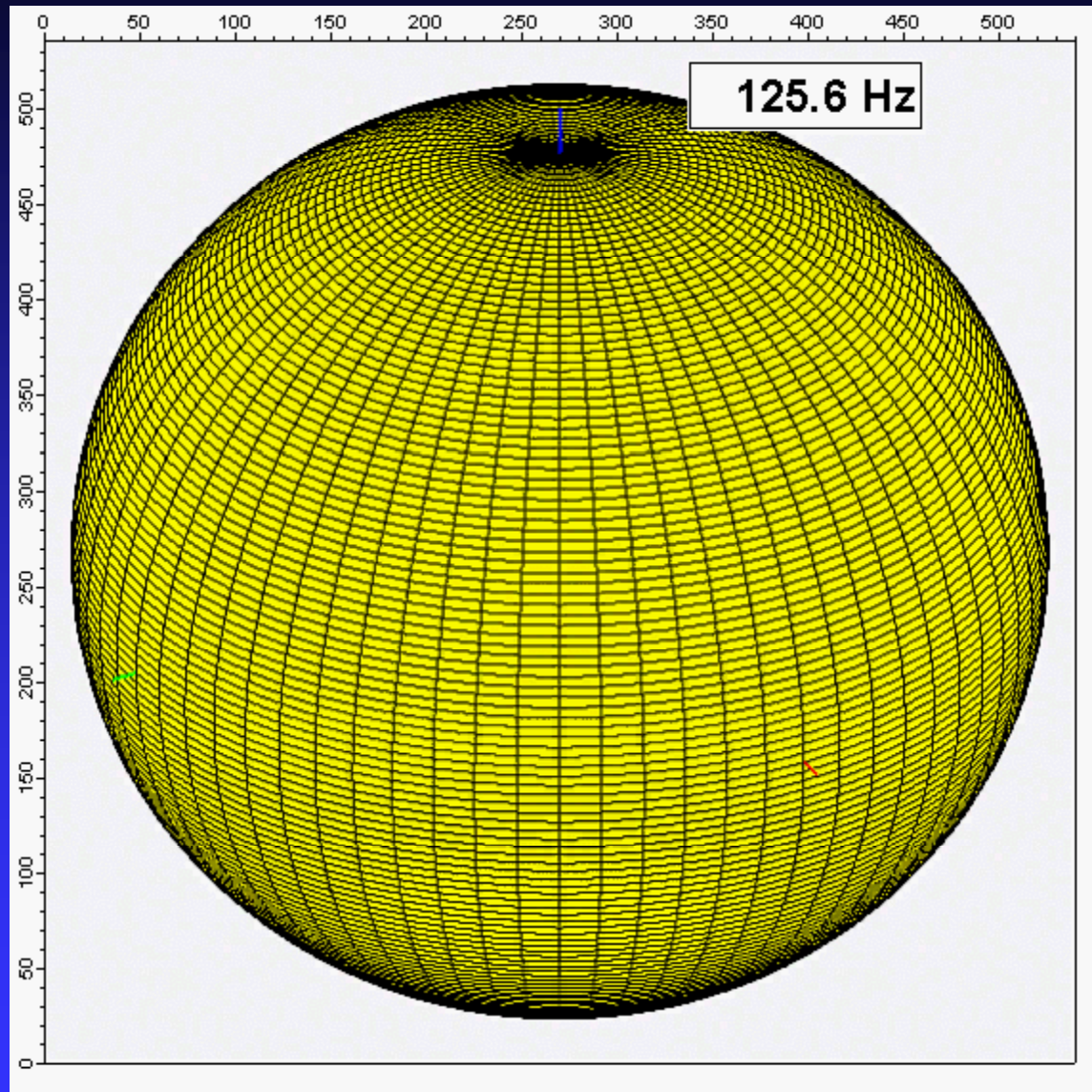




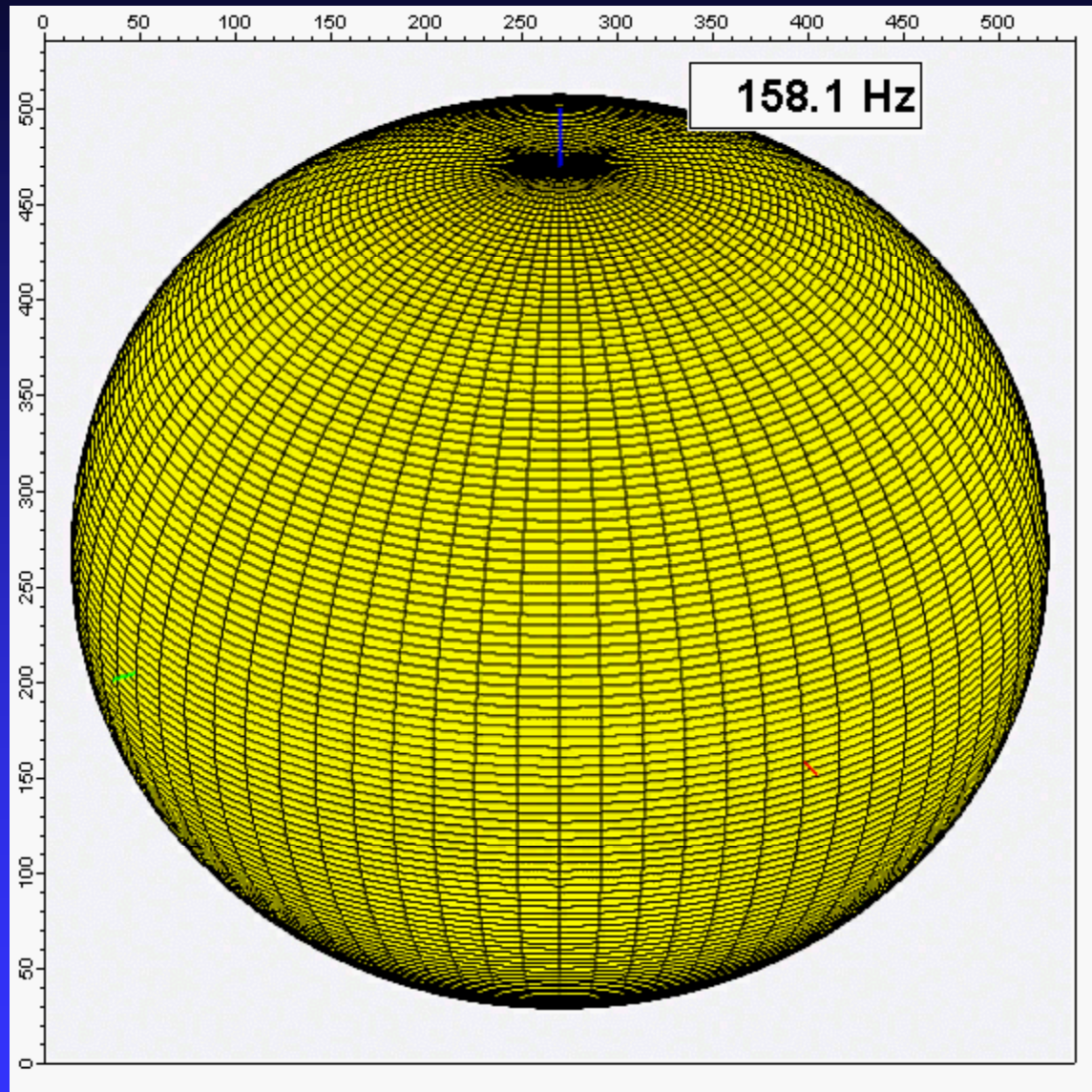
# CBT Curved Array: Narrow Vertical Coverage Cont.



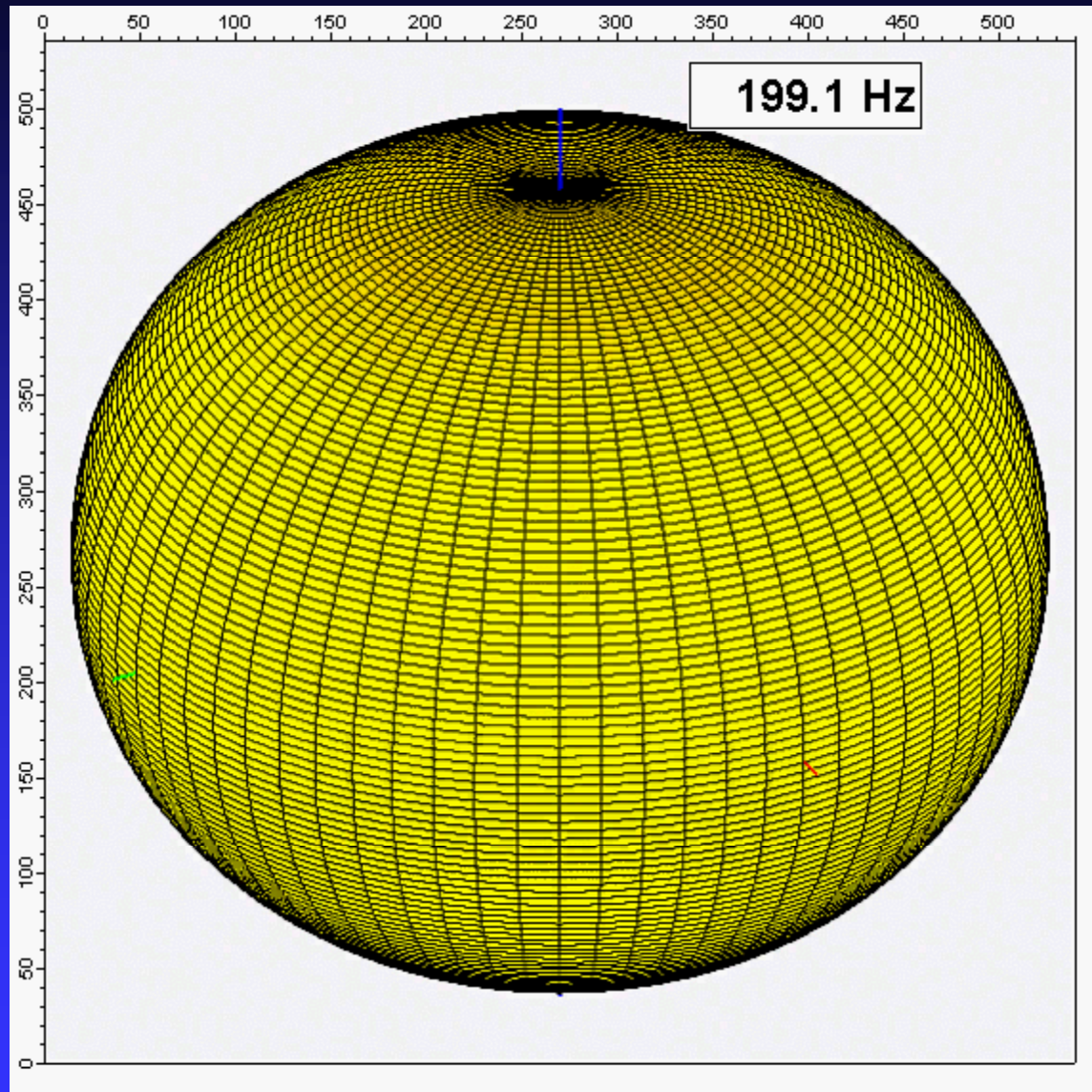
# CBT Curved Array: Narrow Vertical Coverage Cont.



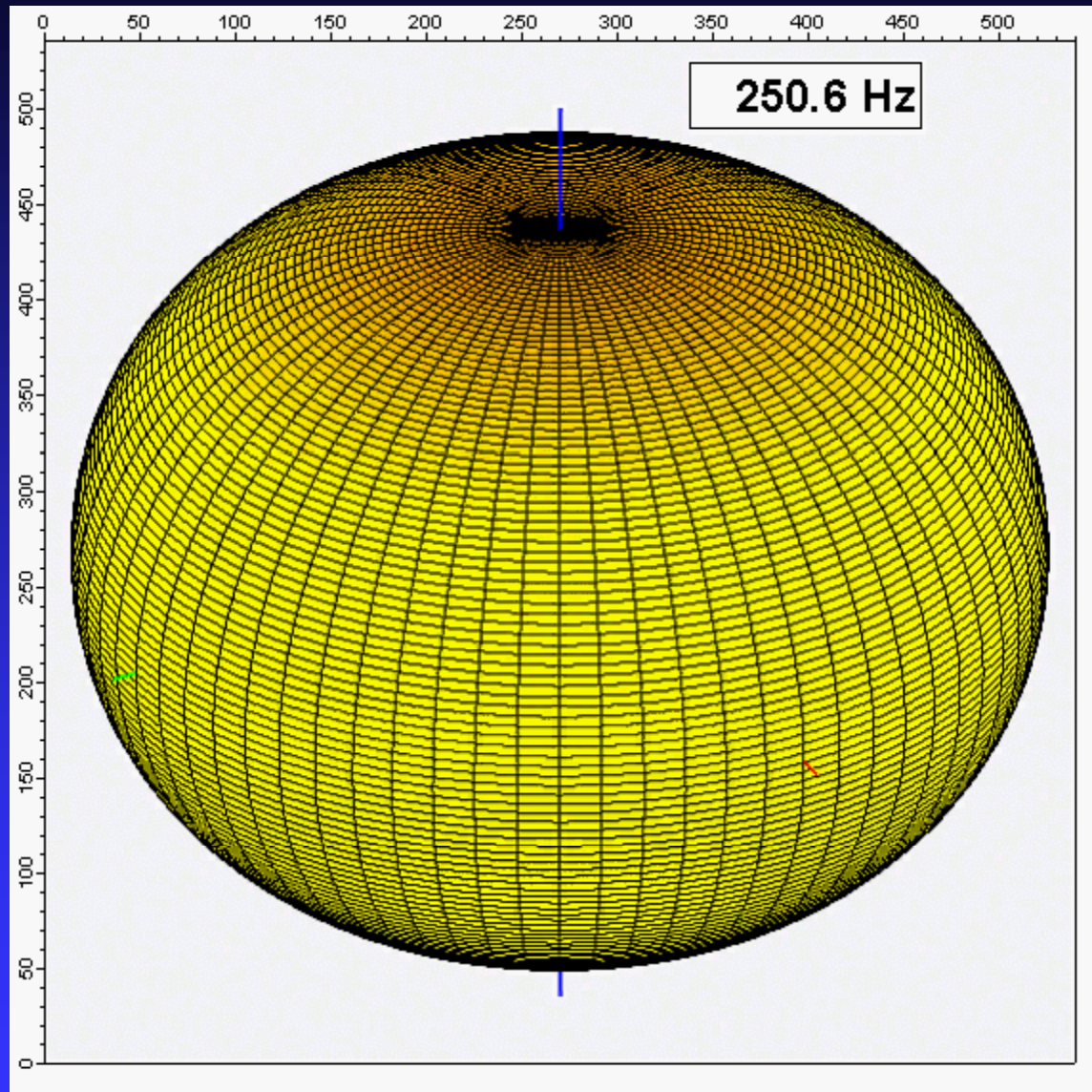
# CBT Curved Array: Narrow Vertical Coverage Cont.



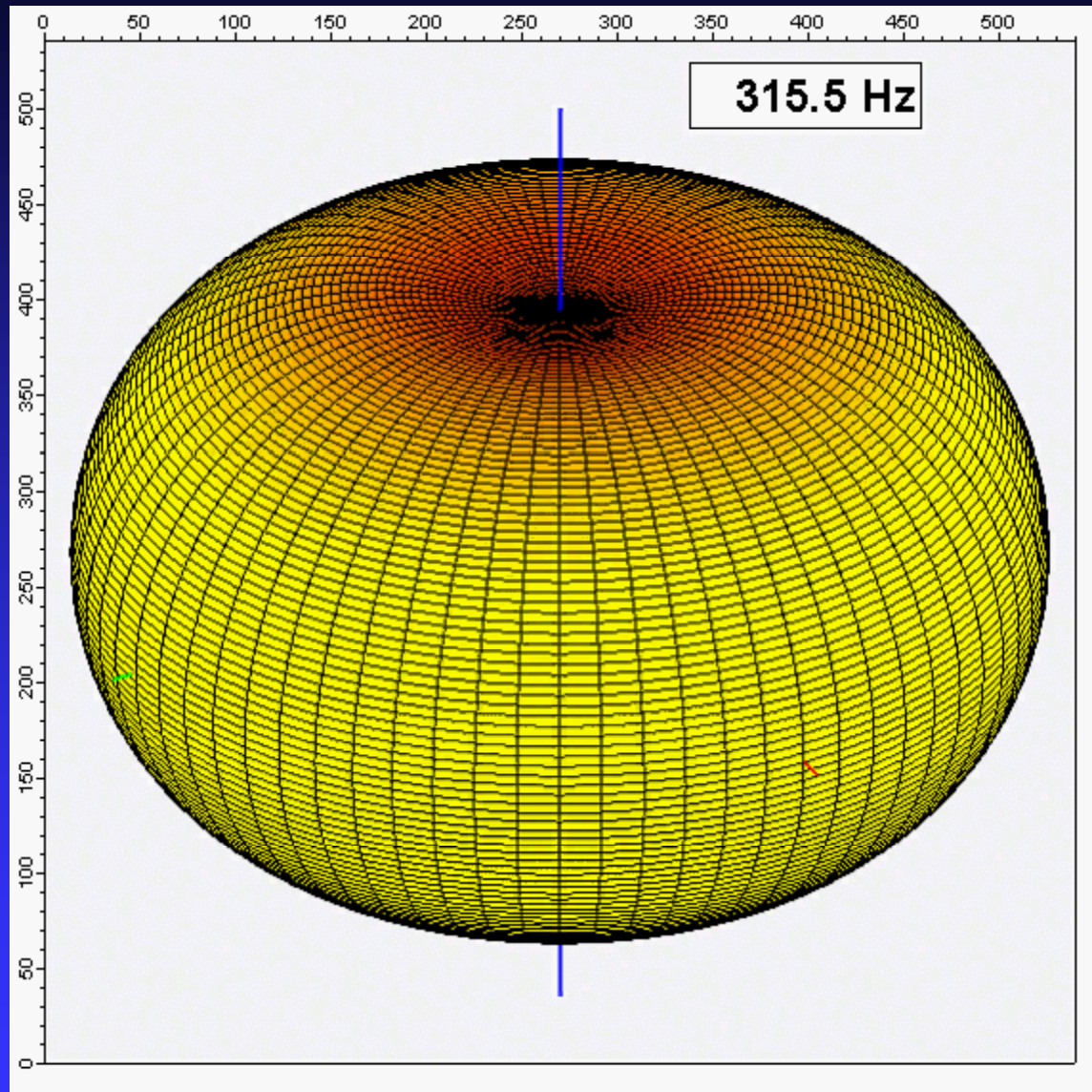
# CBT Curved Array: Narrow Vertical Coverage Cont.



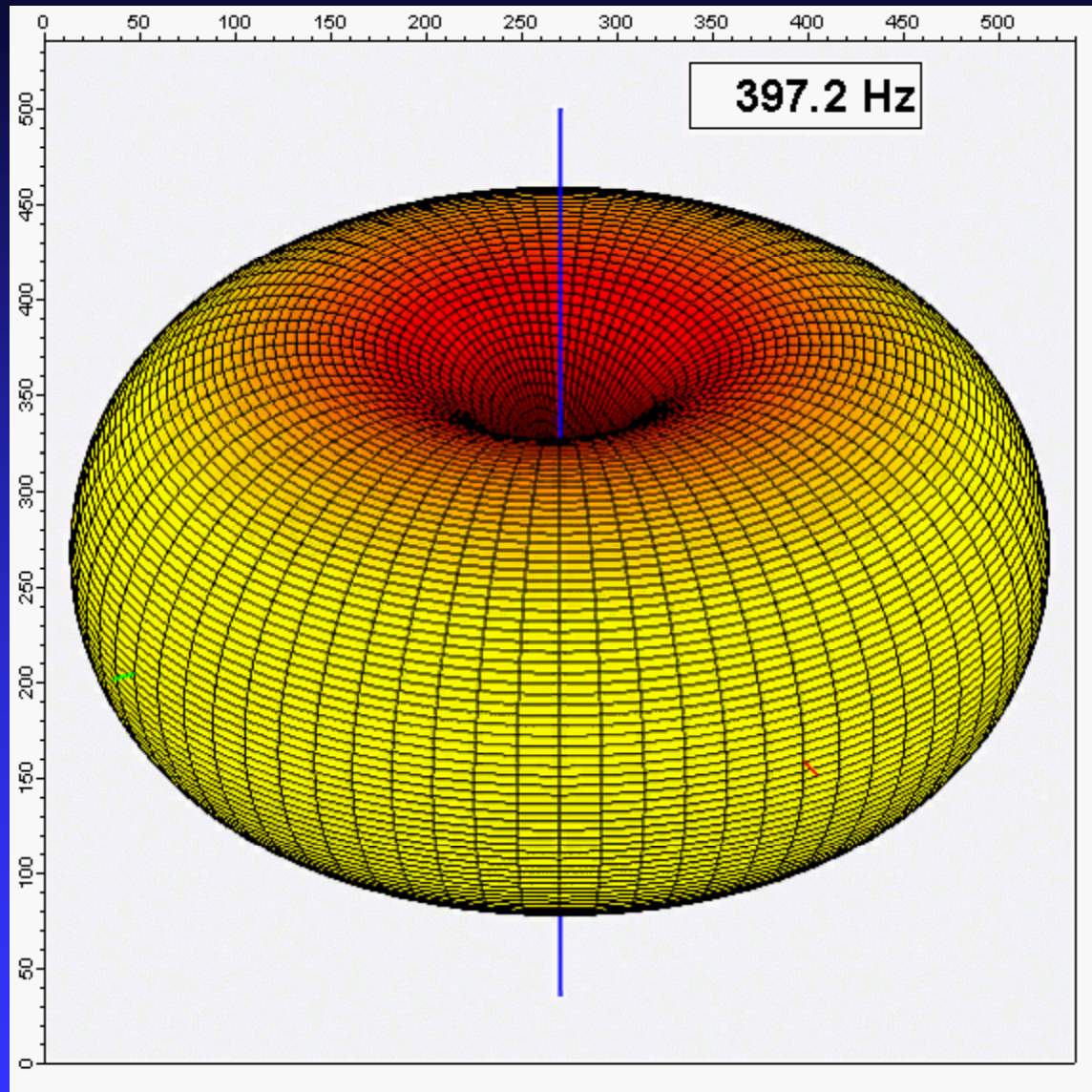
# CBT Curved Array: Narrow Vertical Coverage Cont.



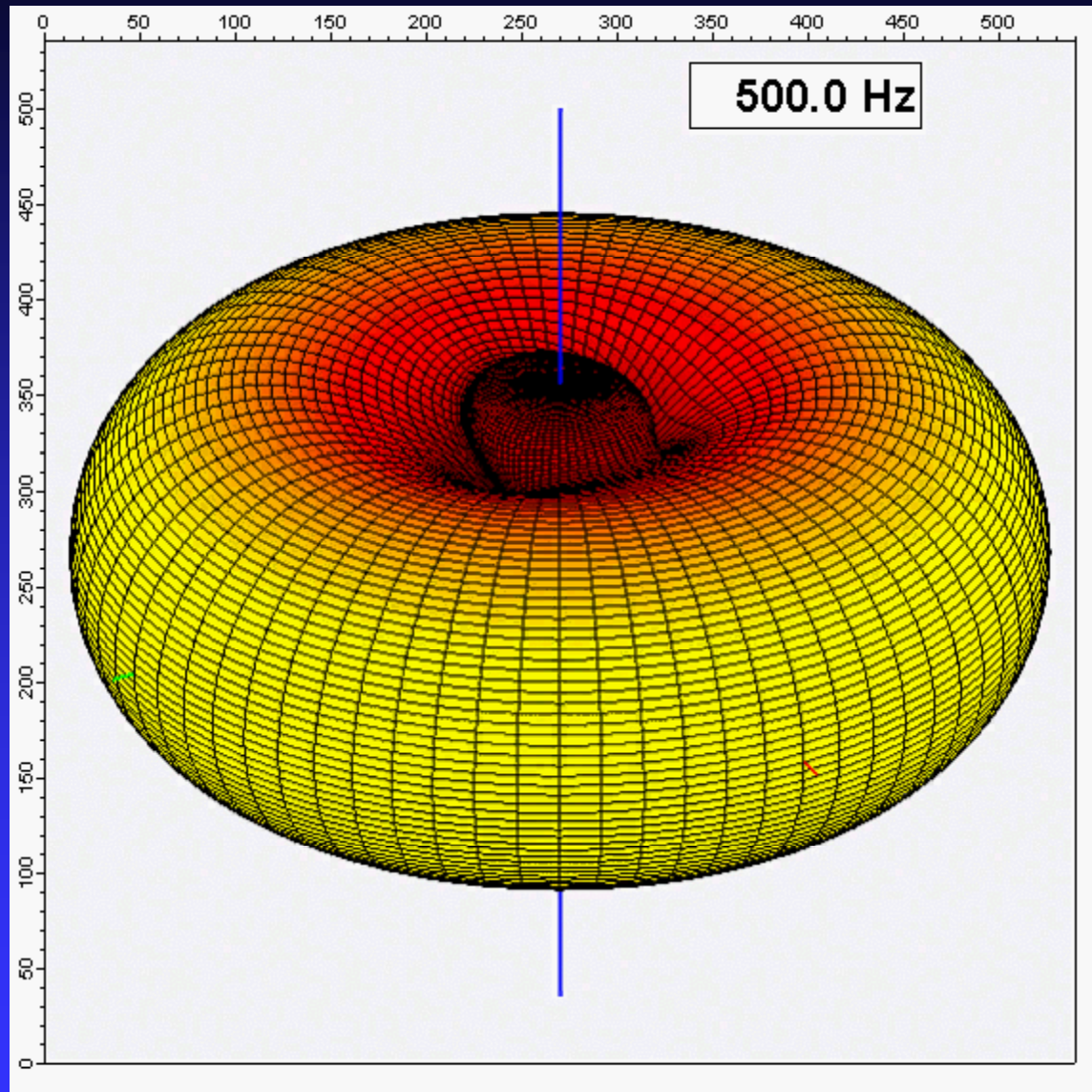
# CBT Curved Array: Narrow Vertical Coverage Cont.



# CBT Curved Array: Narrow Vertical Coverage Cont.

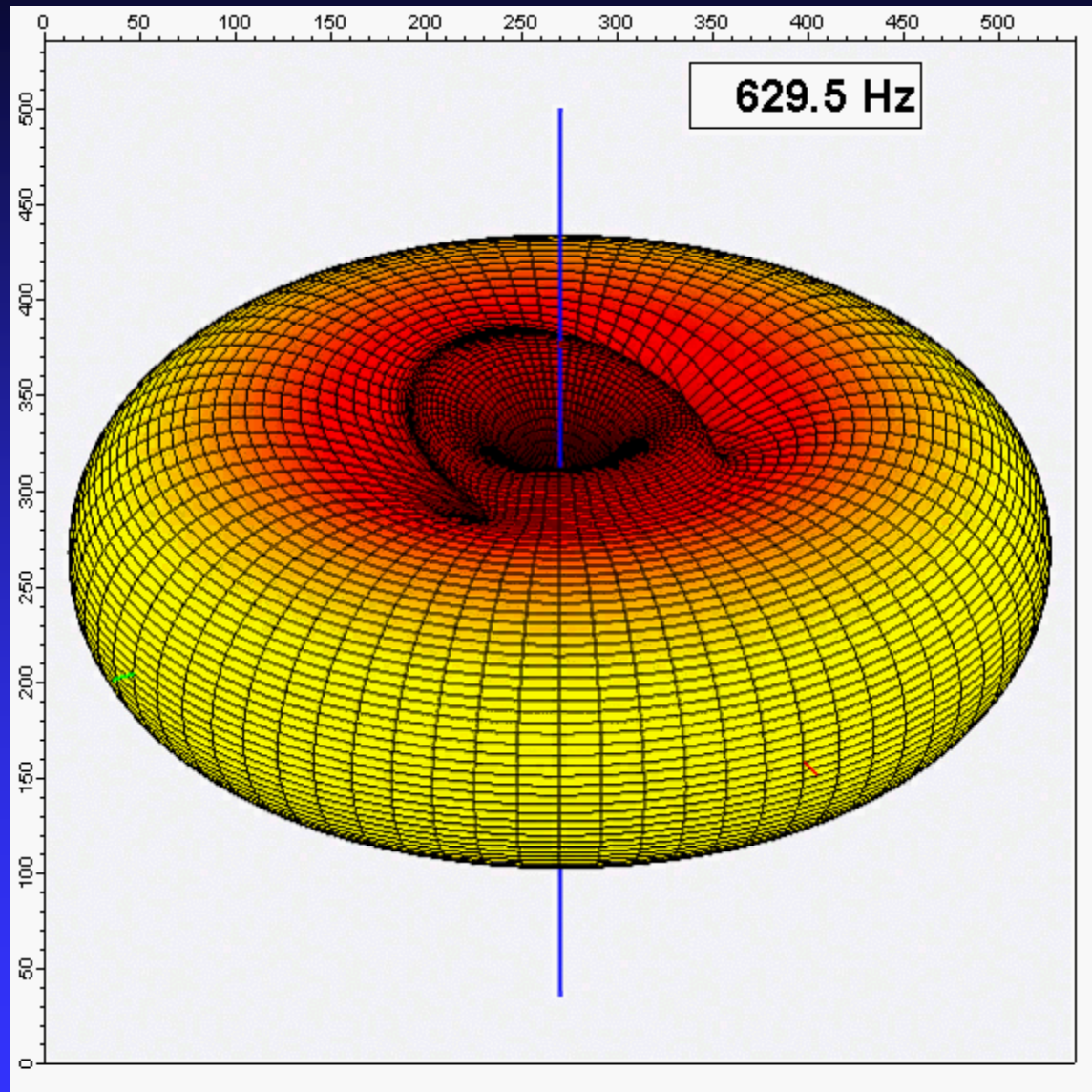


# CBT Curved Array: Narrow Vertical Coverage Cont.

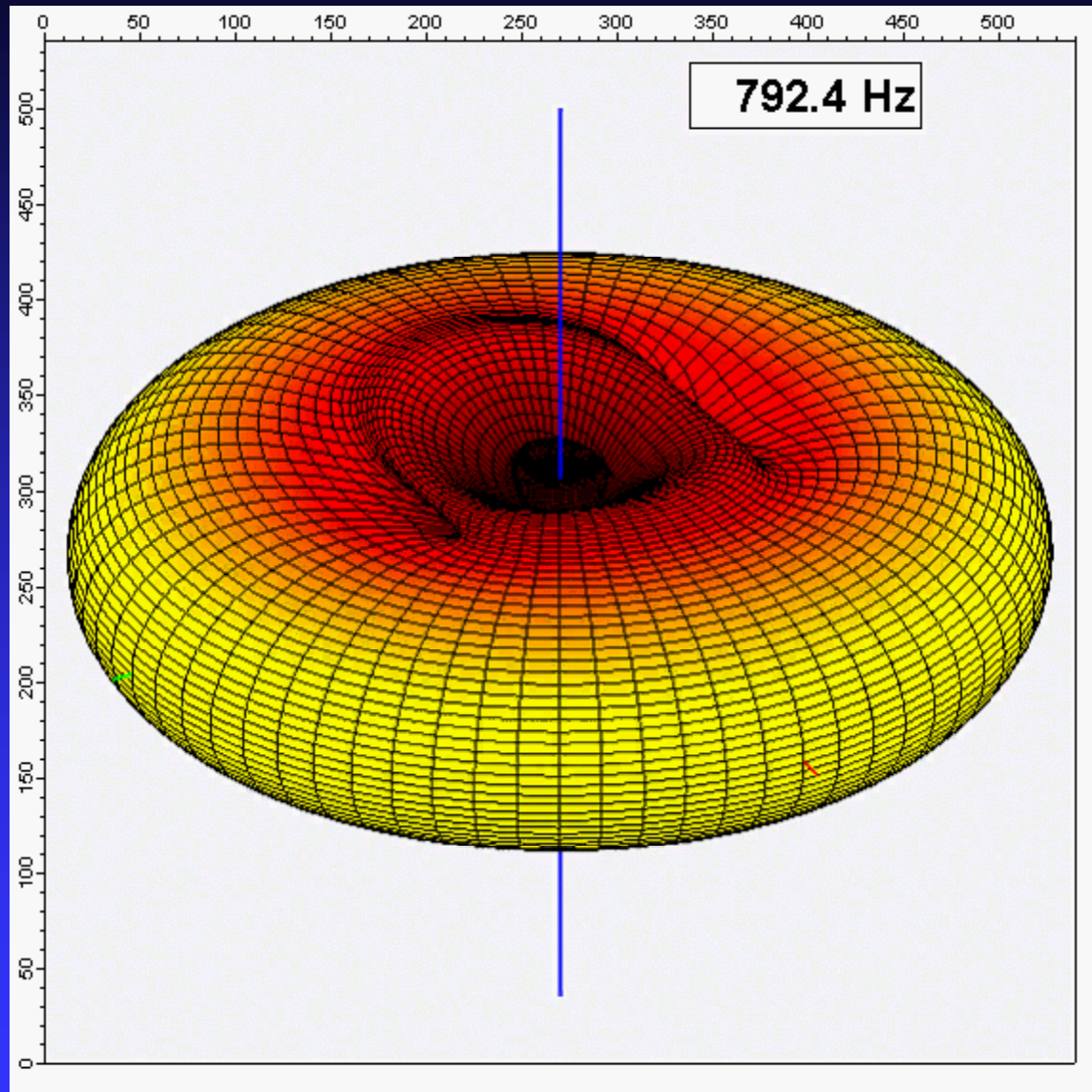




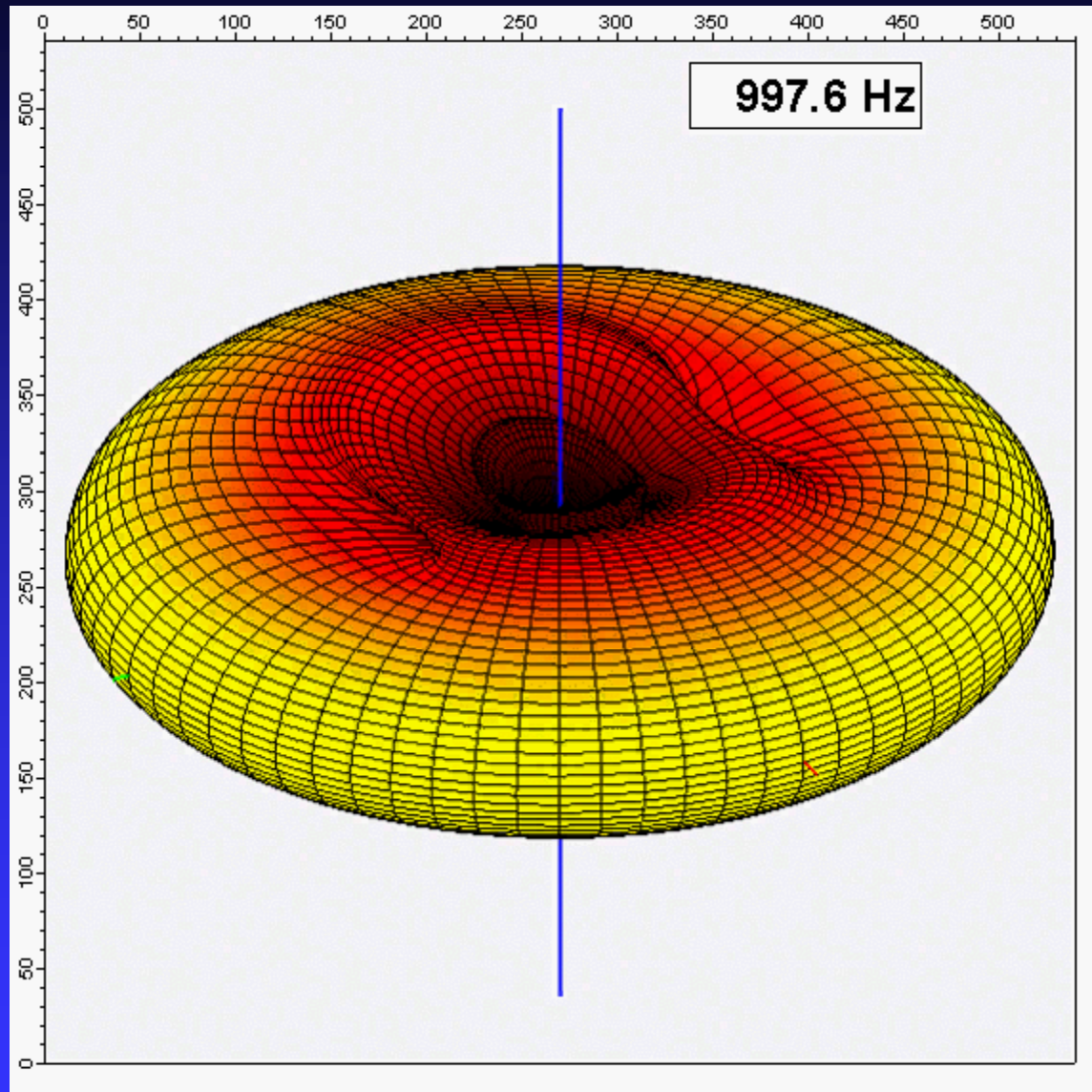
# CBT Curved Array: Narrow Vertical Coverage Cont.



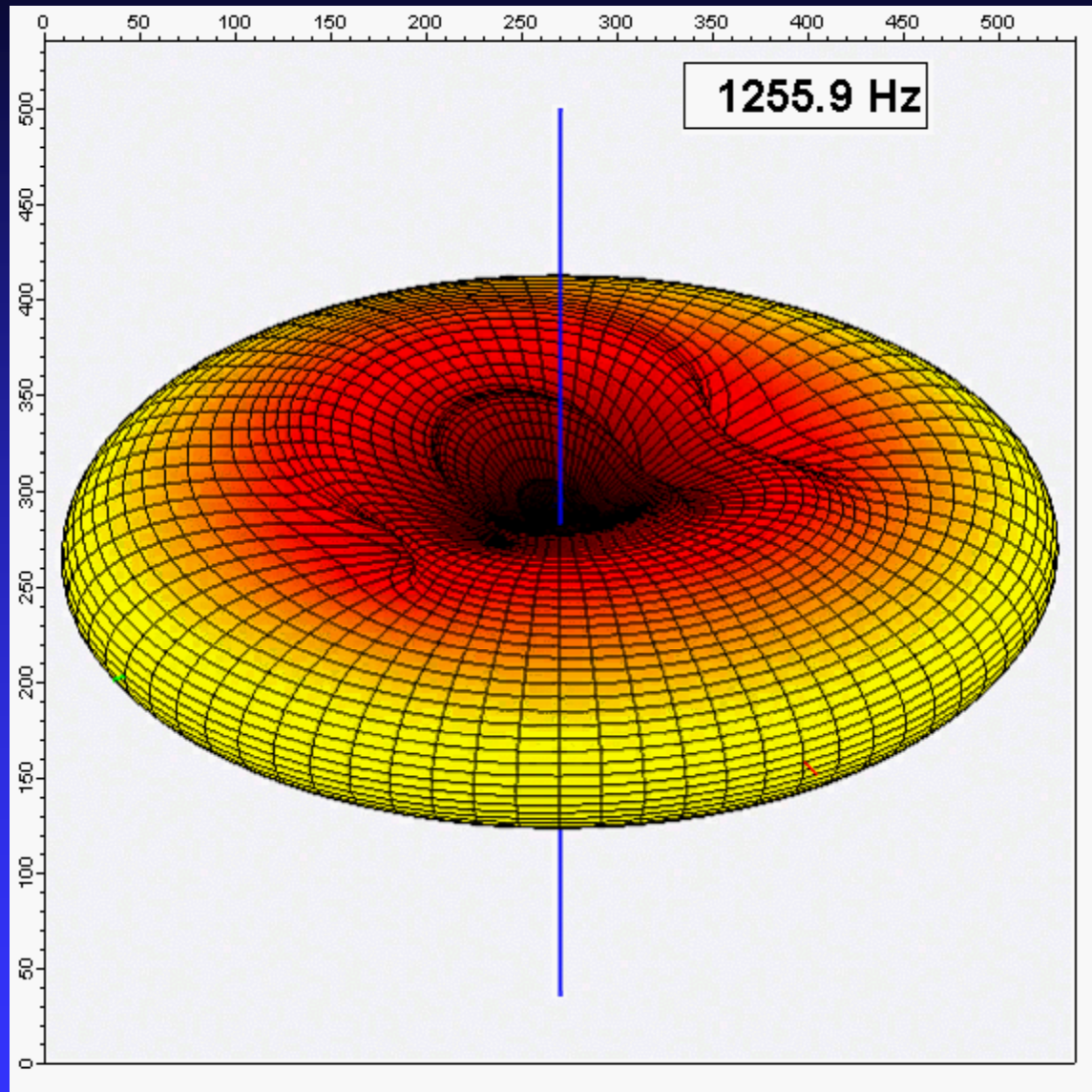
# CBT Curved Array: Narrow Vertical Coverage Cont.



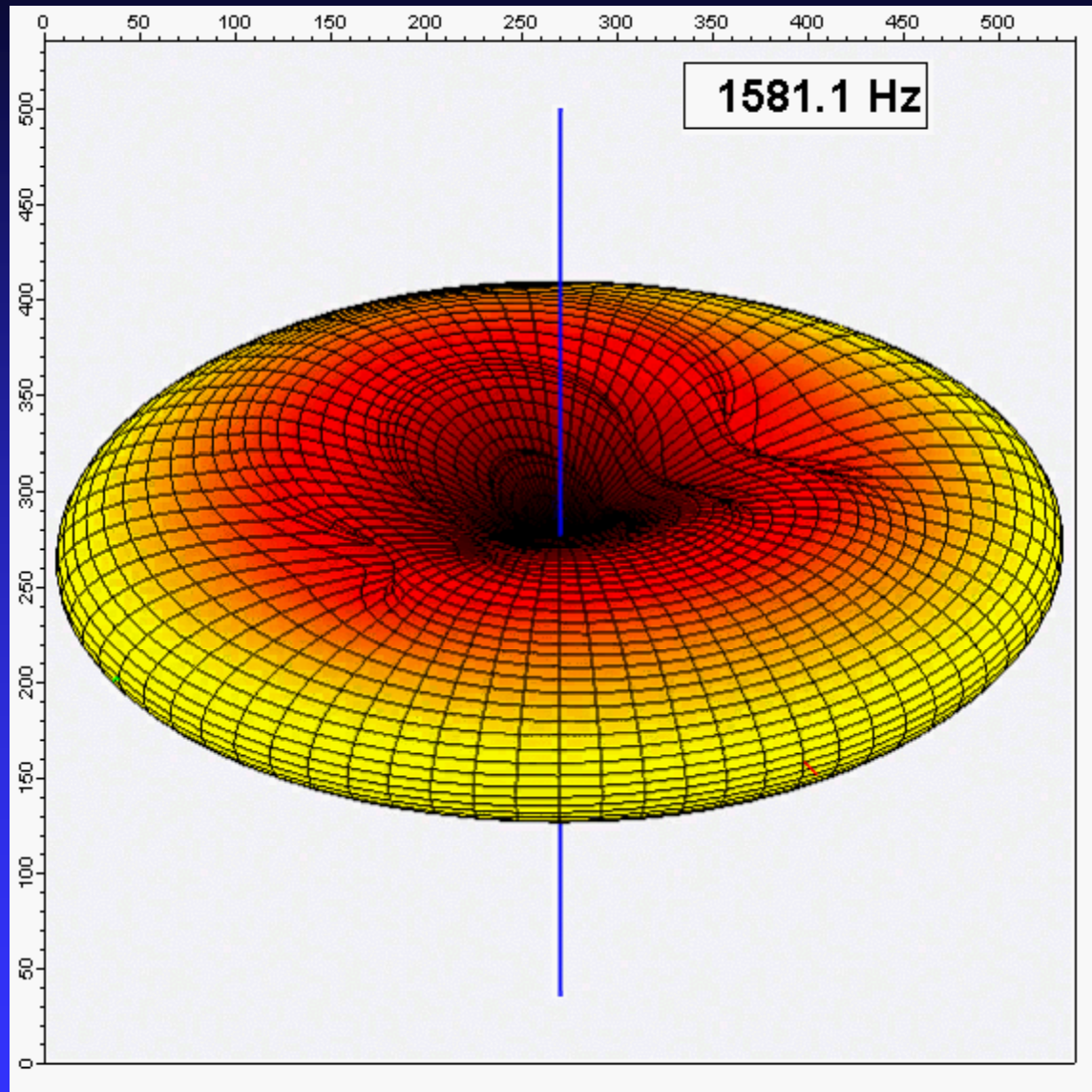
# CBT Curved Array: Narrow Vertical Coverage Cont.



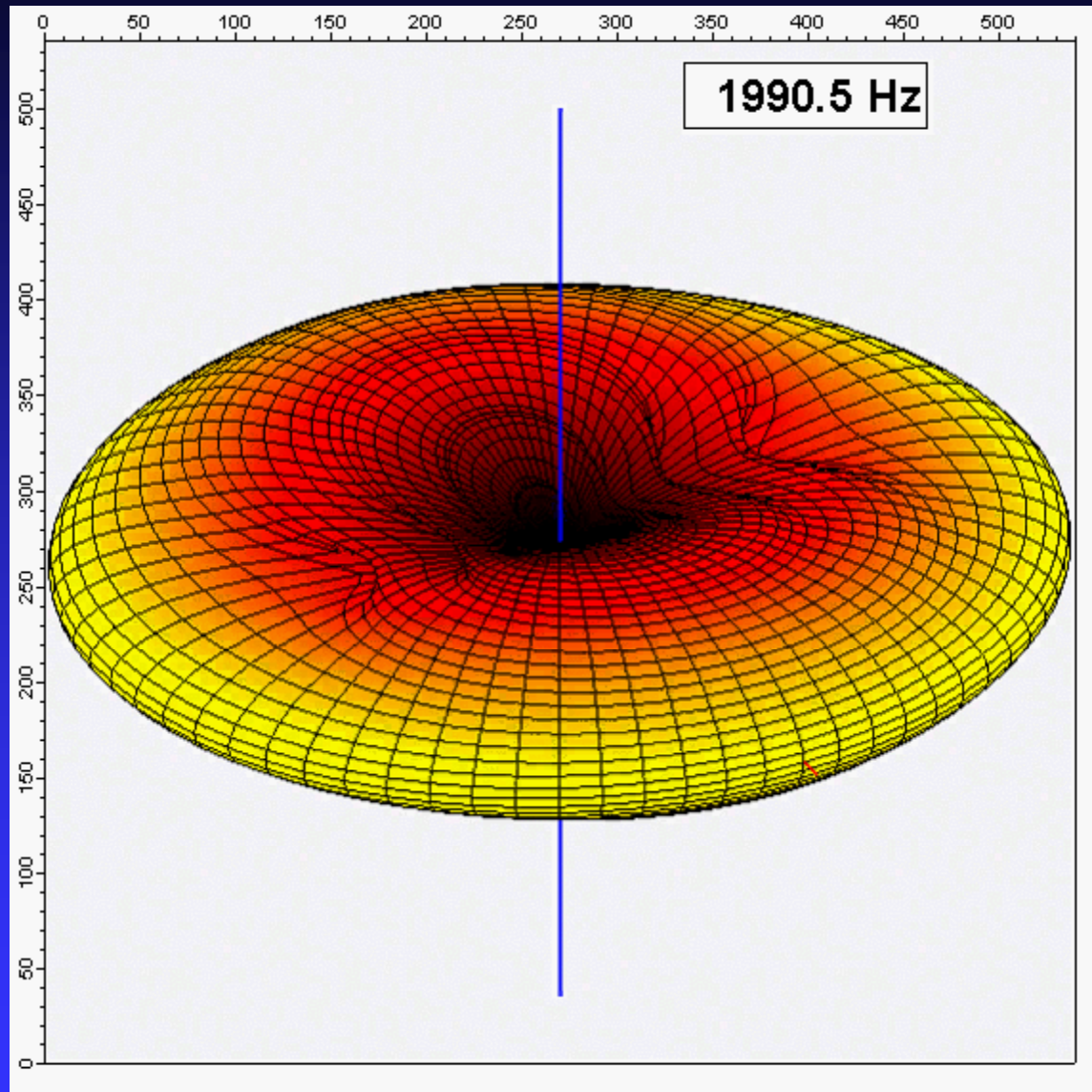
# CBT Curved Array: Narrow Vertical Coverage Cont.



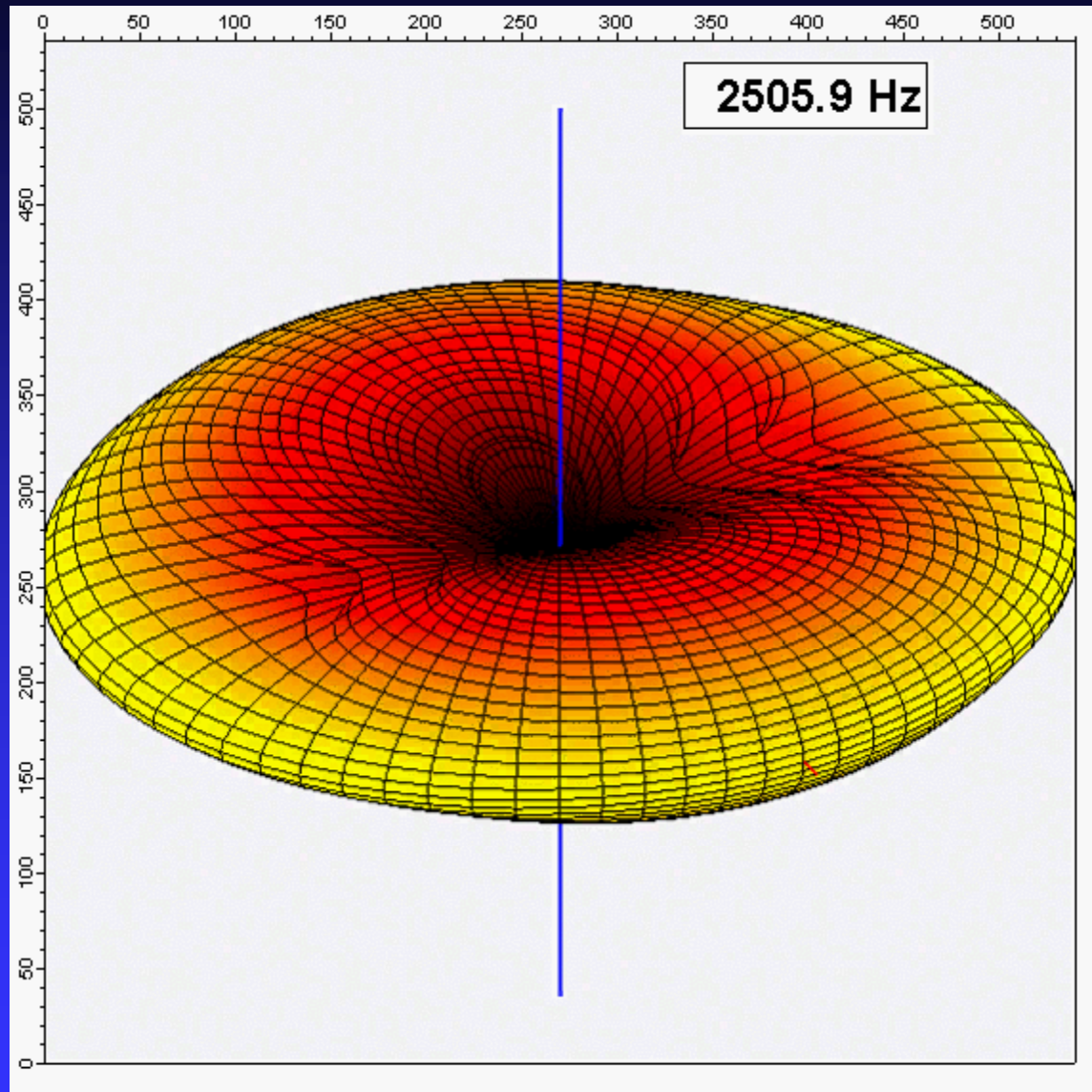
# CBT Curved Array: Narrow Vertical Coverage Cont.



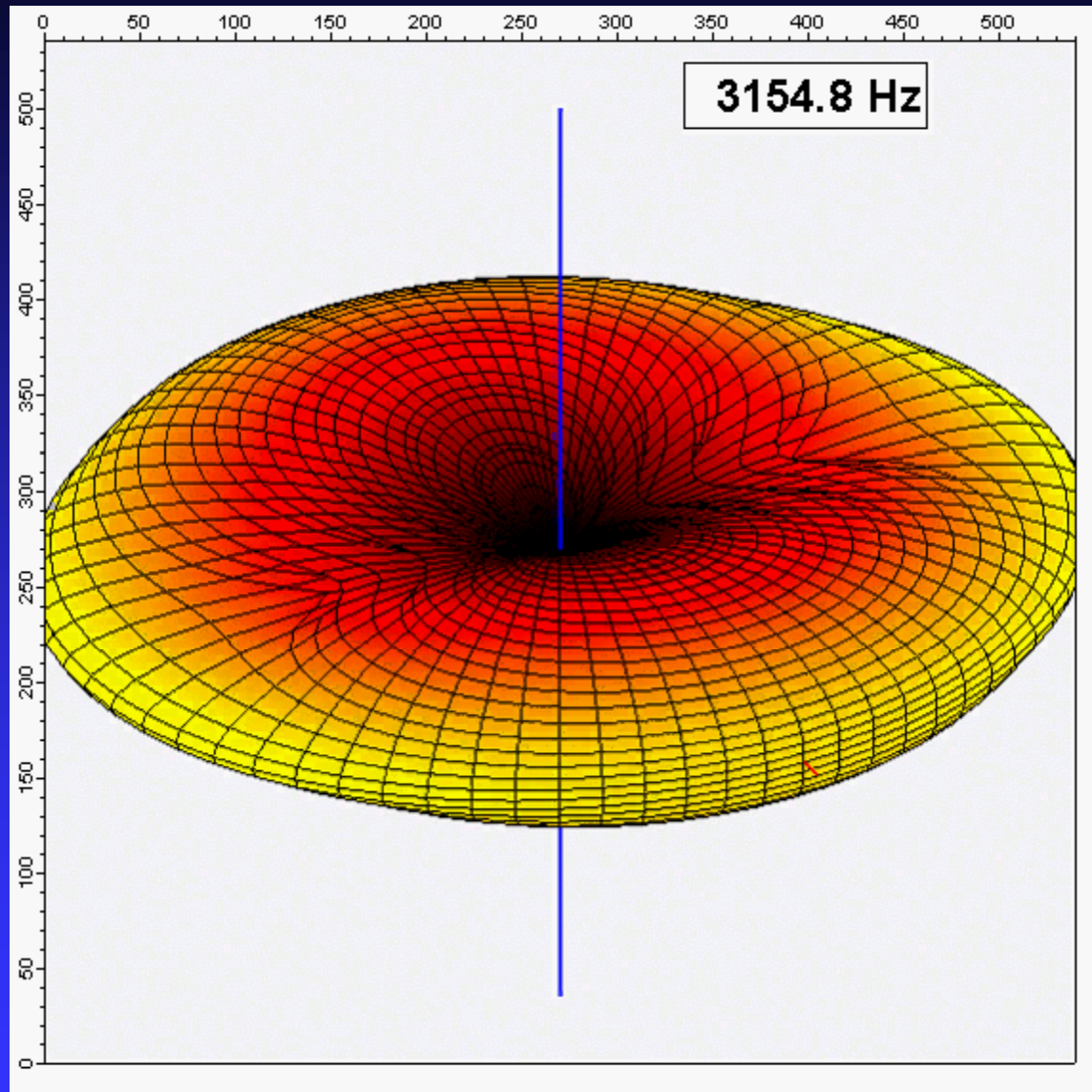
# CBT Curved Array: Narrow Vertical Coverage Cont.



# CBT Curved Array: Narrow Vertical Coverage Cont.

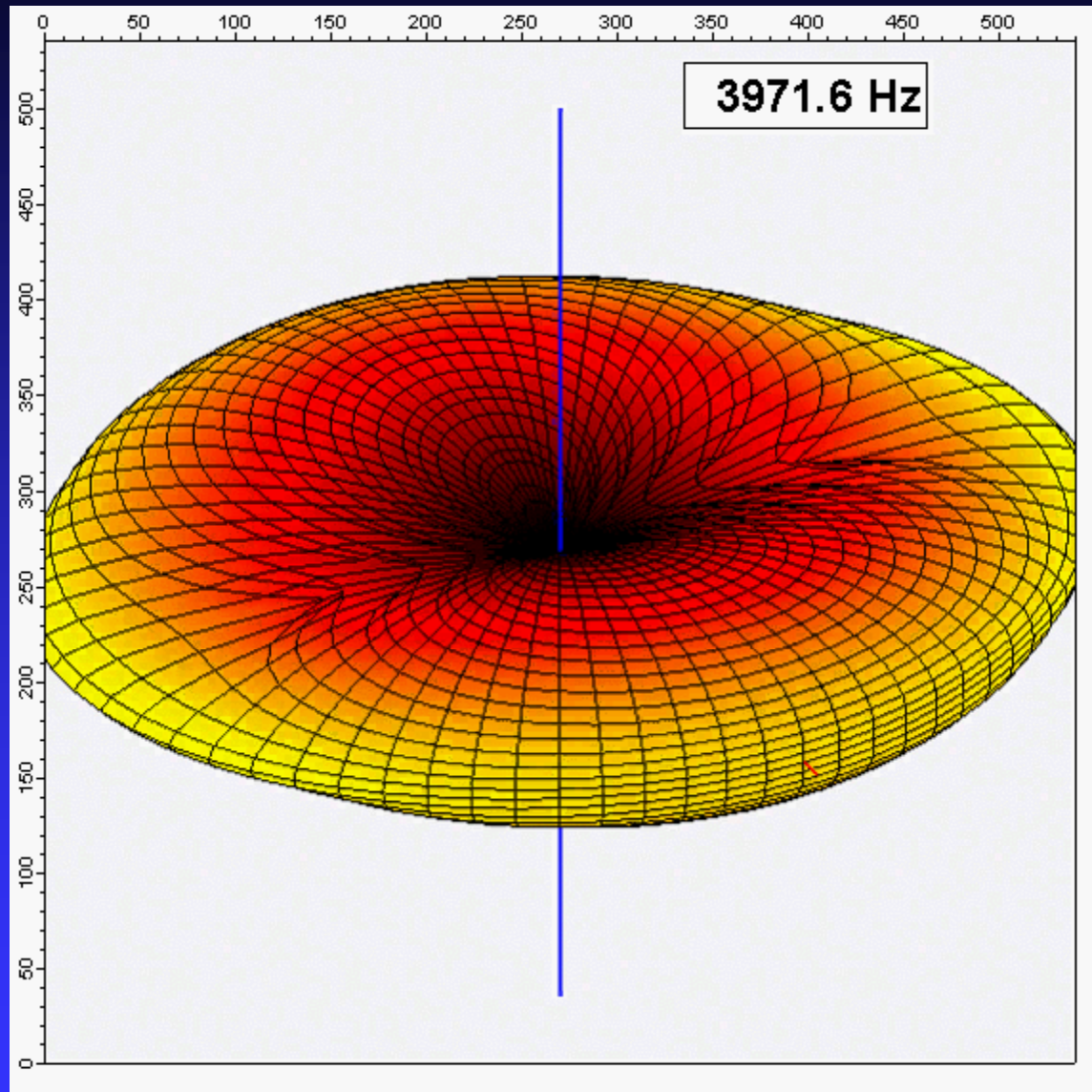


# CBT Curved Array: Narrow Vertical Coverage Cont.

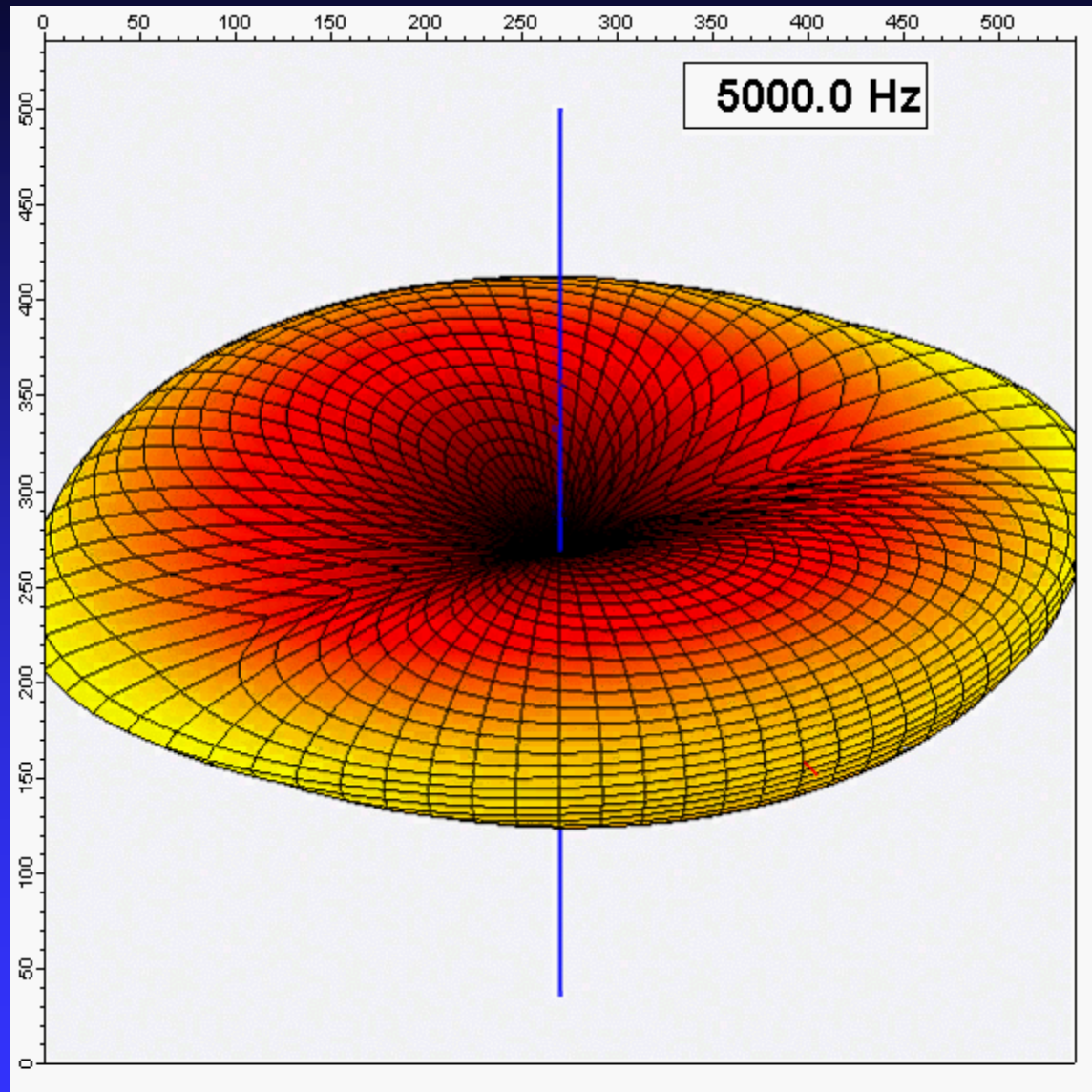




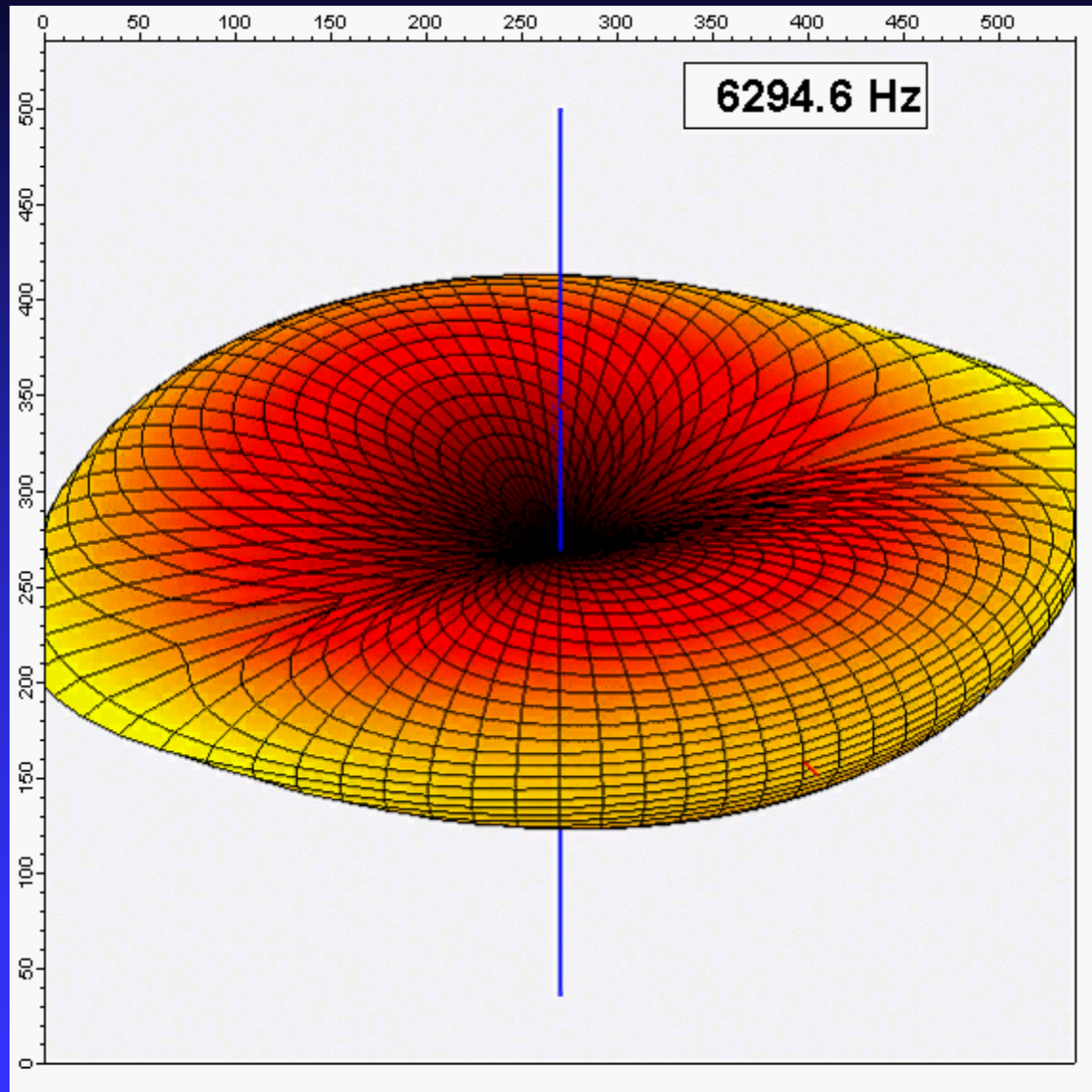
# CBT Curved Array: Narrow Vertical Coverage Cont.



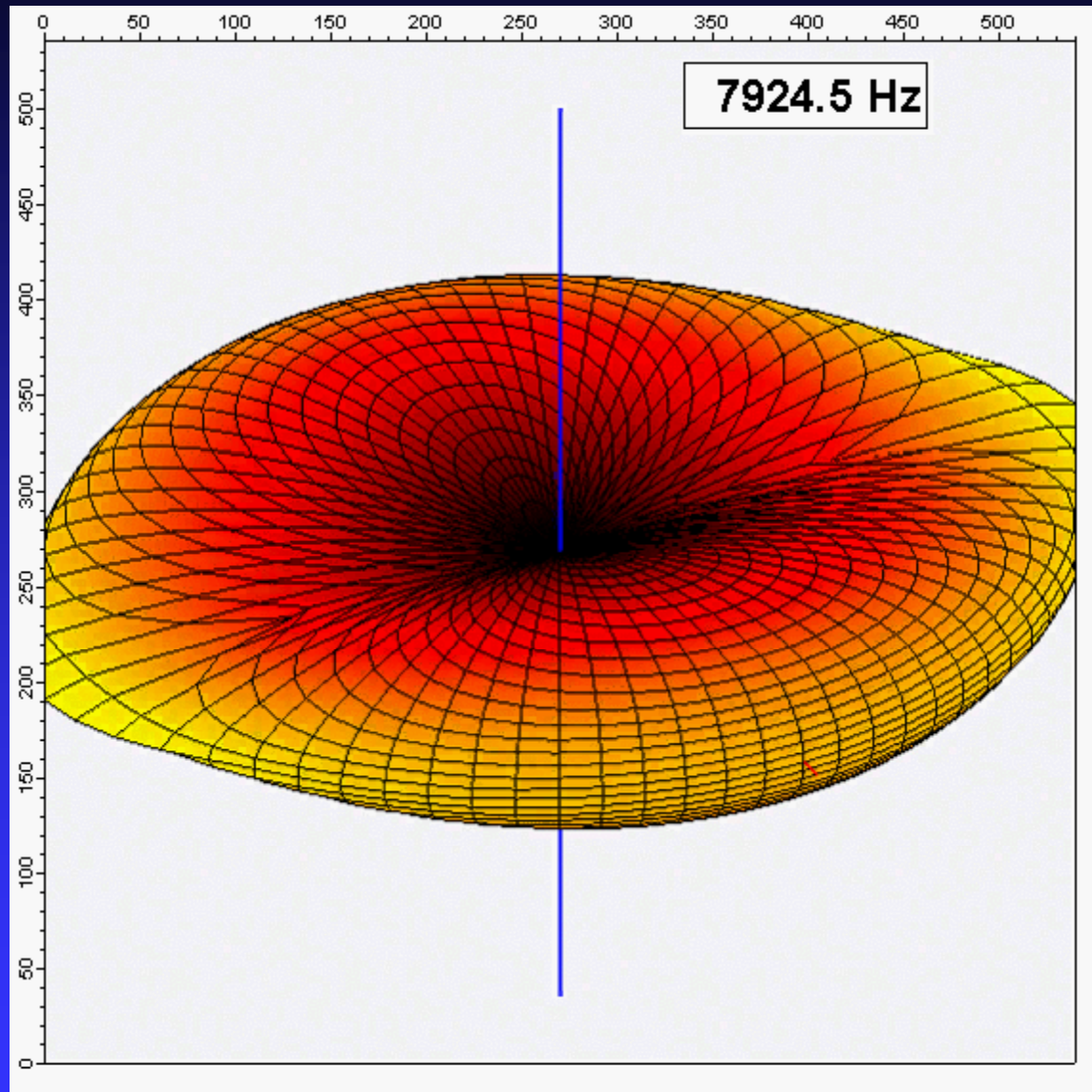
# CBT Curved Array: Narrow Vertical Coverage Cont.



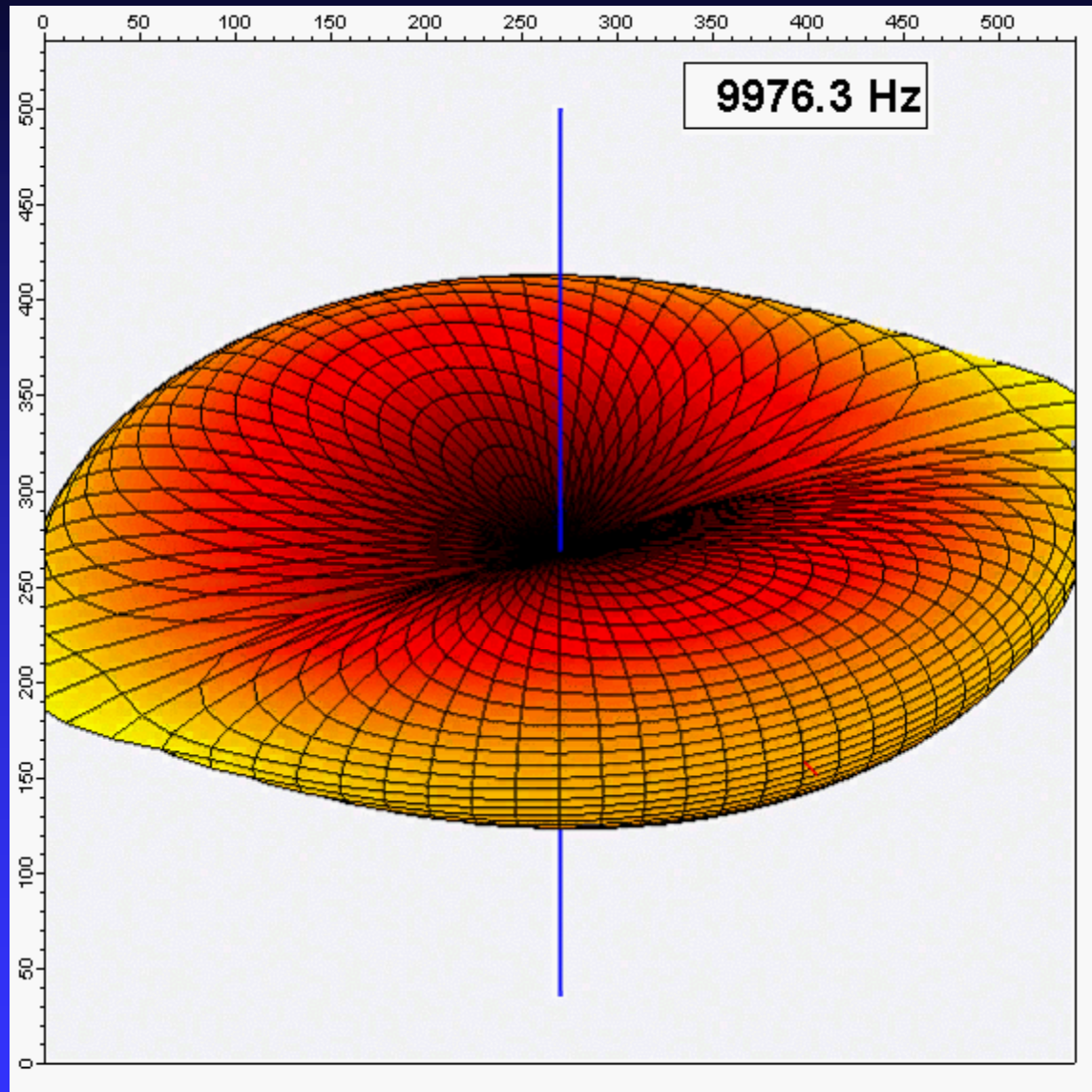
# CBT Curved Array: Narrow Vertical Coverage Cont.



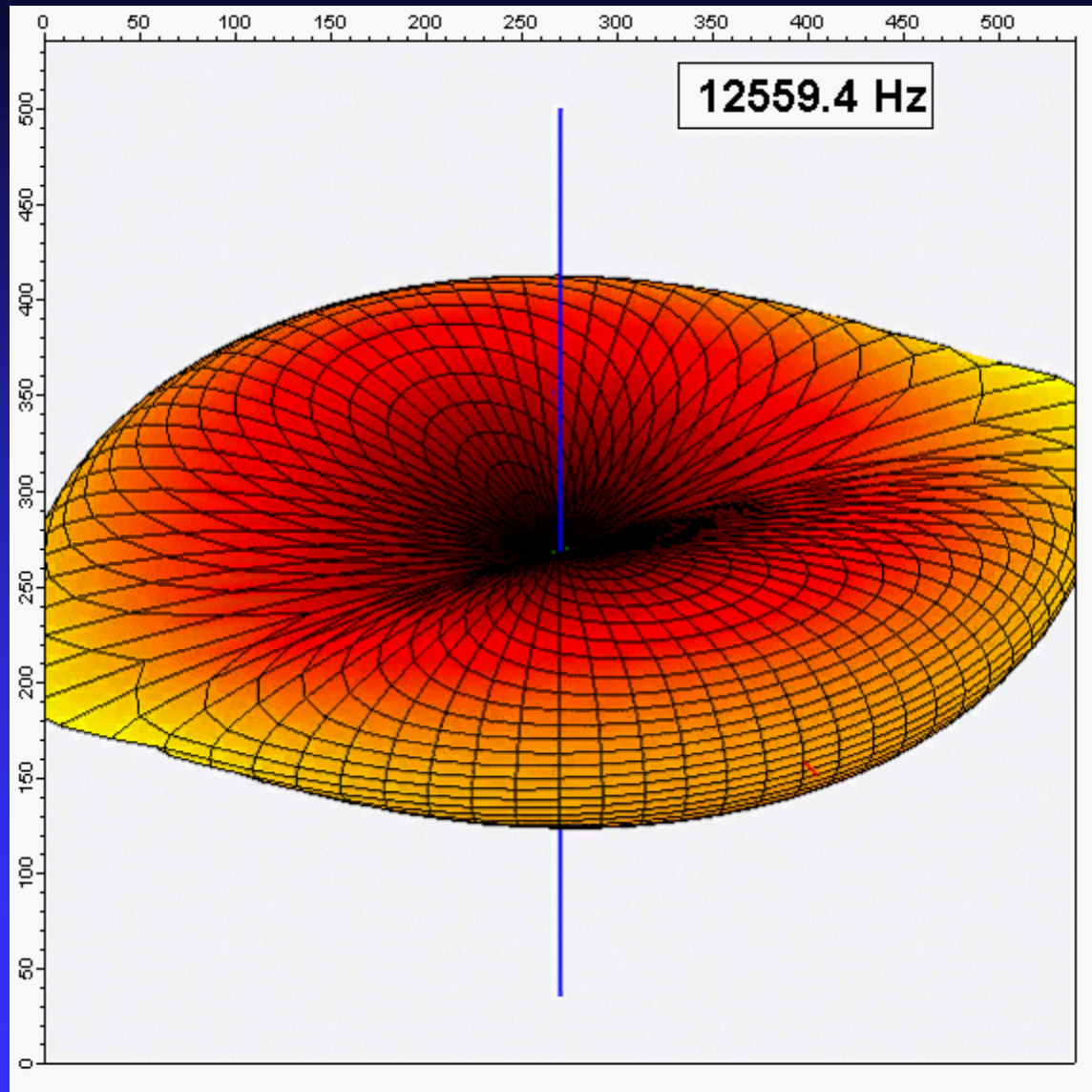
# CBT Curved Array: Narrow Vertical Coverage Cont.



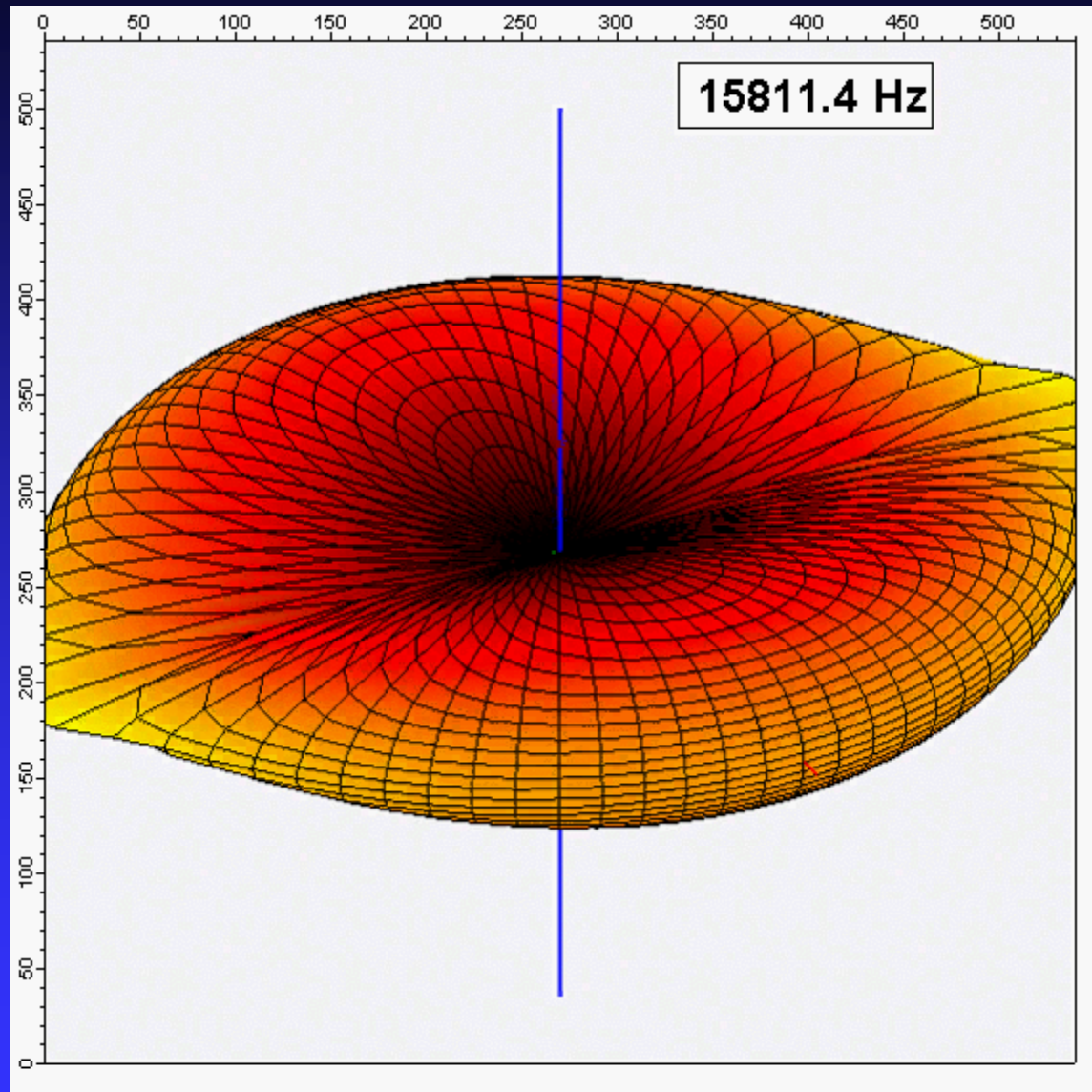
# CBT Curved Array: Narrow Vertical Coverage Cont.



# CBT Curved Array: Narrow Vertical Coverage Cont.



# CBT Curved Array: Narrow Vertical Coverage Cont.

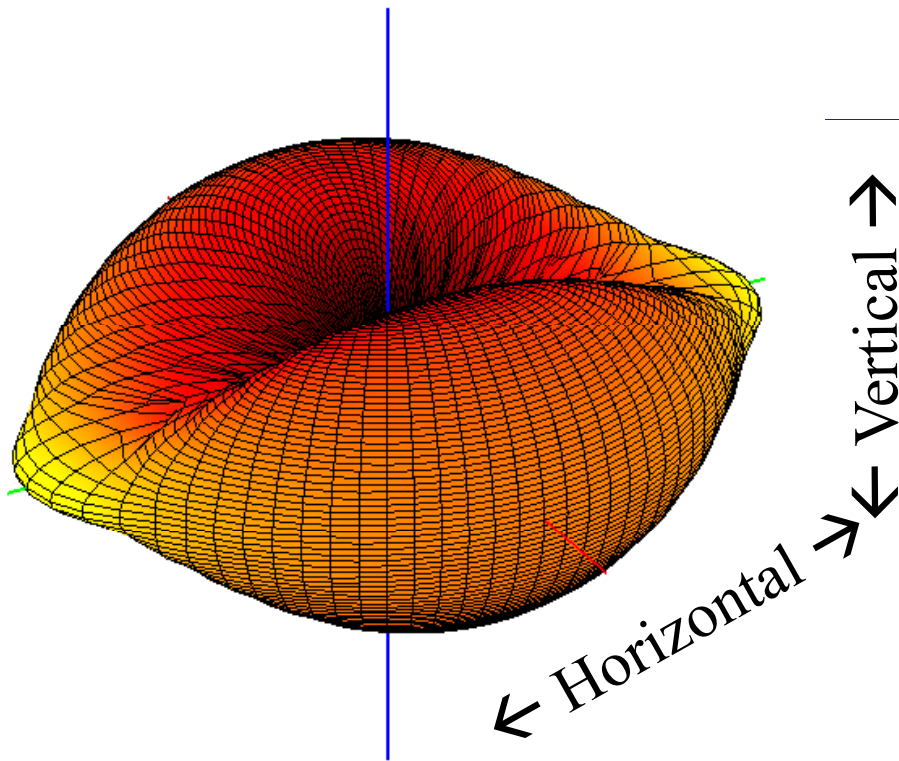


## CBT Curved Array: Narrow Vertical Coverage Cont.

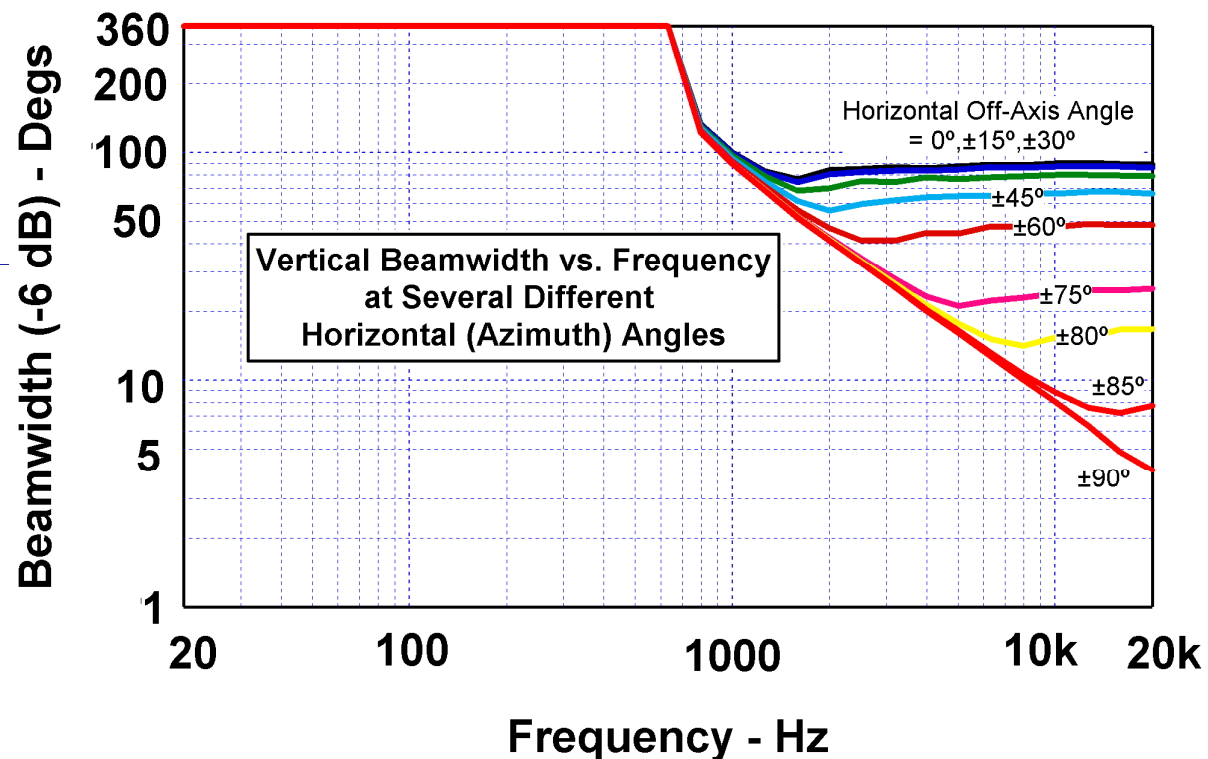
# End of CBT Narrow Vertical Coverage Movie



# Variation of CBT Curved-Line Array Vertical Beamwidth with Horizontal (Azimuth) Angle



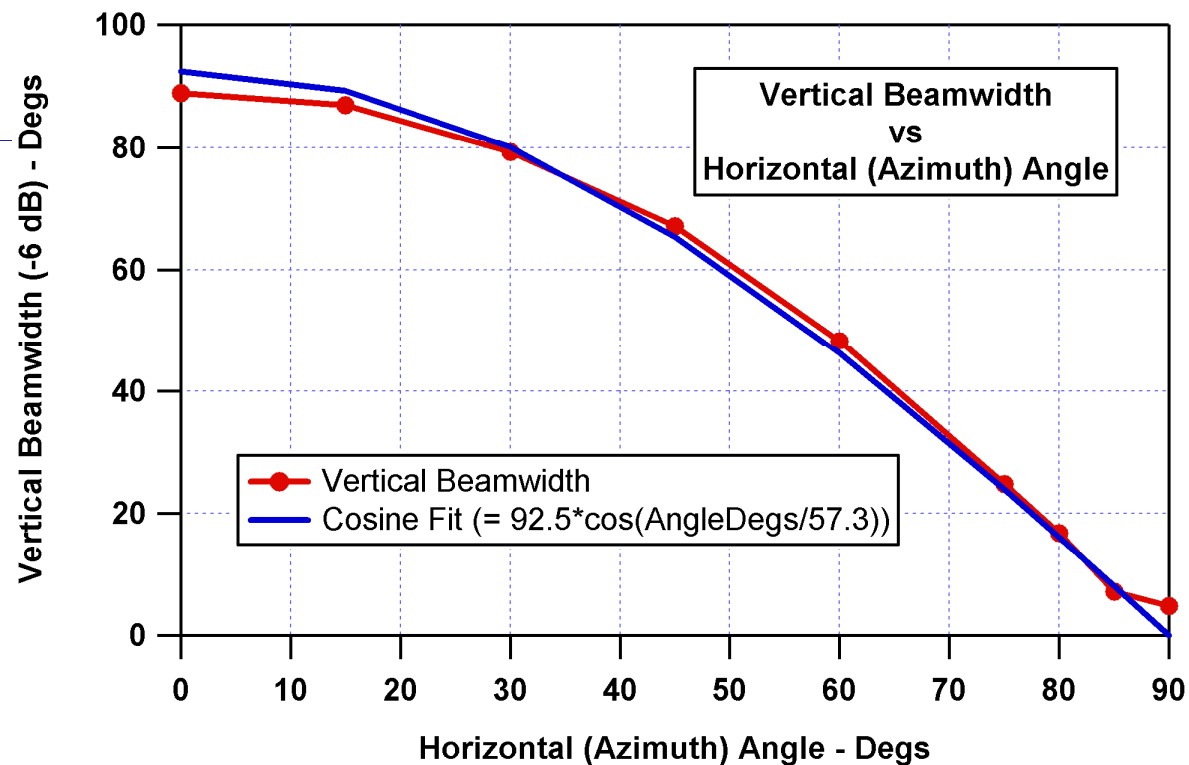
# Variation of CBT Curved-Line Array Vertical Beamwidth with Horizontal (Azimuth) Angle: Cont.



Note that the vertical beamwidth of the array narrows as you go off axis horizontally. Note also that the vertical beamwidth only narrows to an asymptote that represents the beamwidth vs. frequency of a straight-line array of the same height as the CBT array.

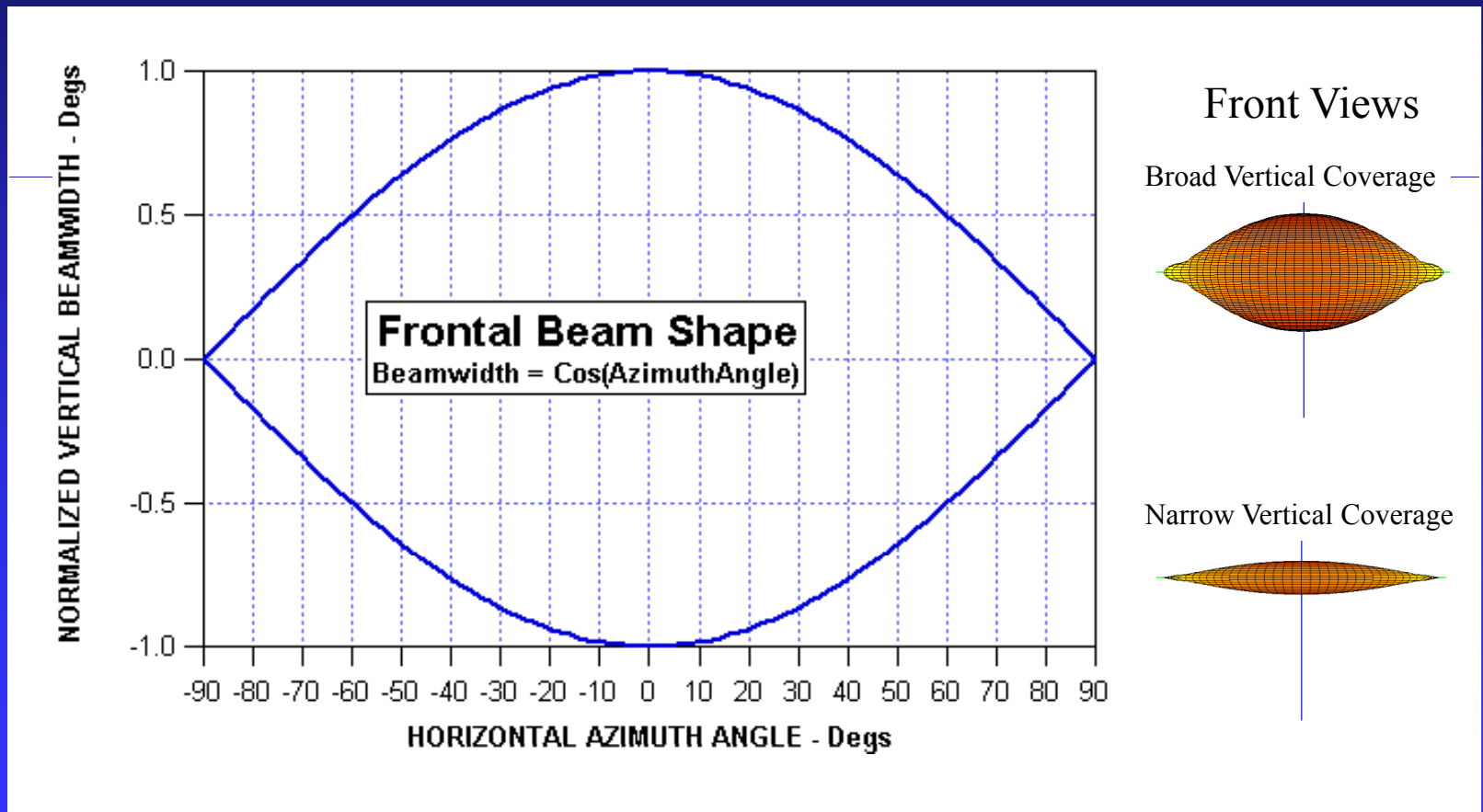
# Variation of CBT Curved-Line Array Vertical Beamwidth with Horizontal (Azimuth) Angle: Cont.

## Best Fit Cosine Variation



# Variation of CBT Curved-Line Array Vertical Beamwidth with Horizontal (Azimuth) Angle: Cont.

## Eyeball Shape



# The Full-Sphere Sound Field of CBT Loudspeaker Line Arrays: Conclusions:

- This paper has shown that the radiation pattern of the curved-line CBT array is completely un-like the pattern of a conventional straight-line array that exhibits controlled vertical coverage but completely un-controlled omni-directional horizontal coverage with a pattern that is symmetric around the vertical axis.
- Although still a line source, the CBT curved-line array exhibits significant horizontal directivity in addition to its expected vertical control. This horizontal directivity is exhibited by its variation of vertical beamwidth with horizontal off-axis angle.
- The vertical beamwidth varies smoothly from full value on axis to a low minimum value at right angles to the arrays forward axis in a manner that follows the cosine of the off-axis horizontal angle.

# The Full-Sphere Sound Field of CBT Loudspeaker Line Arrays: Conclusions: Cont.

- Interestingly, the curved-line array provides its maximum intensity at right angles ( $\pm 90^\circ$  off-axis horizontally) to its primary defined axis. This is because all the sources that make up the array are essentially equidistant from the observation point and hence in phase at these angles.
- This feature however, does not upset the controlled and smooth frontal coverage and directivity of the array because these regions of highest level are confined to very small regions on the sides of the radiation pattern.

# The Full-Sphere Sound Field of CBT Loudspeaker Line Arrays: Conclusions: Cont.

- The CBT curved-line array provides surprisingly constant directivity and uniform vertical beamwidth above a frequency related to the size of the array and its vertical coverage angle.
- This is accomplished without any complicated frequency-dependent signal processing. The only processing required is simple frequency-independent gain adjustment of the levels of the individual drivers to accomplish the Legendre shading.

Thanks for  
listening!