# Ground-Plane Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays

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# Overview

- Introduction
- A Brief CBT History and Overview
- Steps to Create a Ground-Plane Array from a Free-Standing Array
- Sound-Field Simulations
- Measurements: Frequency Response Over the Ground Plane
- Conclusions: Advantages of CBT Ground-Plane Array

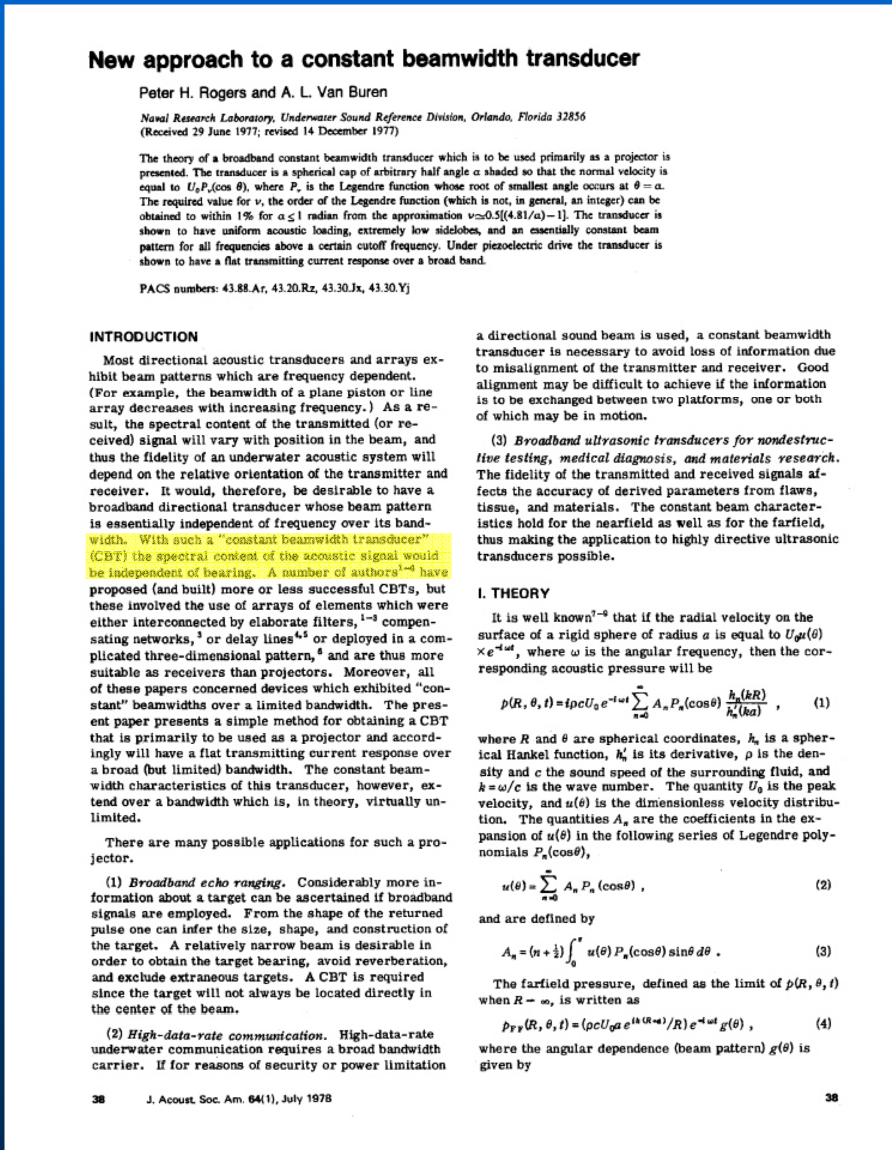
# Overview of Constant Beamwidth Transducer Theory

- First formulated in JASA papers published in 1978 and 1983 describing underwater transducers based on shaded spherical caps (U.S. Naval Research Labs)
  - P. H. Rogers, and A. L. Van Buren, "New Approach to a Constant Beamwidth Transducer," J. Acous. Soc. Am., vol. 64, no. 1, pp. 38-43 (1978 July).
  - A. L. Van Buren, L. D. Luker, M. D. Jevnager, and A. C. Tims, "Experimental Constant Beamwidth Transducer," J. Acous. Soc. Am., vol. 73, no. 6, pp. 2200-2209 (1983 June).

- Applied to loudspeaker arrays in four AES papers by Keele in 2000, 2002, and 2003
  - [1] "The Application of Broadband Constant Beamwidth Transducer (CBT) Theory to Loudspeaker Arrays," presented at the 109th convention of the Audio Engineering Society, Preprint 5216 (Sept. 2000).
  - [2] "Implementation of Straight-Line and Flat-Panel Constant Beamwidth Transducer (CBT) Loudspeaker Arrays Using Signal Delays," 113th Convention of the Audio Engineering Society, Preprint 5653 (Oct. 2002).
  - [3] "The Full-Sphere Sound Field of Constant Beamwidth Transducer (CBT) Loudspeaker Line Arrays," J. Aud. Eng. Soc., vol. 51, no. 7/8., pp. 611-624 (July/August 2003).
  - [4] "Practical Implementation of Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays," presented at the 115th Convention of the Audio Engineering Society, New York (Oct. 2003).

# Rogers & Van Buren 1978

## (Lots of heavy-duty math!)



# New approach to a constant beamwidth transducer

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(Received 29 June 1977; revised 14 December 1977)

The theory of a broadband constant beamwidth transducer which is to be used primarily as a projector is presented. The transducer is a spherical cap of arbitrary half angle $\alpha$ shaded so that the normal velocity is equal to $U_0 P_\nu(\cos\theta)$, where $P_\nu$ is the Legendre function whose root of smallest angle occurs at $\theta = \alpha$. The required value for $\nu$, the order of the Legendre function (which is not, in general, an integer) can be obtained to within 1% for $\alpha \leq 1$ radian from the approximation $\nu \simeq 0.5[(4.81/\alpha)-1]$. The transducer is shown to have uniform acoustic loading, extremely low sidelobes, and an essentially constant beam pattern for all frequencies above a certain cutoff frequency. Under piezoelectric drive the transducer is shown to have a flat transmitting current response over a broad band.

PACS numbers: 43.88.Ar, 43.20.Rz, 43.30.Jx, 43.30.Yj

## INTRODUCTION

Most directional acoustic transducers and arrays exhibit beam patterns which are frequency dependent. (For example, the beamwidth of a plane piston or line array decreases with increasing frequency.) As a result, the spectral content of the transmitted (or received) signal will vary with position in the beam, and thus the fidelity of an underwater acoustic system will depend on the relative orientation of the transmitter and receiver. It would, therefore, be desirable to have a broadband directional transducer whose beam pattern is essentially independent of frequency over its bandwidth. With such a "constant beamwidth transducer" (CBT) the spectral content of the acoustic signal would be independent of bearing. A number of authors[1–6] have proposed (and built) more or less successful CBTs, but these involved the use of arrays of elements which were either interconnected by elaborate filters,[1–3] compensating networks,[3] or delay lines[4,5] or deployed in a complicated three-dimensional pattern,[6] and are thus more suitable as receivers than projectors. Moreover, all of these papers concerned devices which exhibited "constant" beamwidths over a limited bandwidth. The present paper presents a simple method for obtaining a CBT that is primarily to be used as a projector and accordingly will have a flat transmitting current response over a broad (but limited) bandwidth. The constant beamwidth characteristics of this transducer, however, extend over a bandwidth which is, in theory, virtually unlimited.

There are many possible applications for such a projector.

(1) *Broadband echo ranging.* Considerably more information about a target can be ascertained if broadband signals are employed. From the shape of the returned pulse one can infer the size, shape, and construction of the target. A relatively narrow beam is desirable in order to obtain the target bearing, avoid reverberation, and exclude extraneous targets. A CBT is required since the target will not always be located directly in the center of the beam.

(2) *High-data-rate communication.* High-data-rate underwater communication requires a broad bandwidth carrier. If for reasons of security or power limitation a directional sound beam is used, a constant beamwidth transducer is necessary to avoid loss of information due to misalignment of the transmitter and receiver. Good alignment may be difficult to achieve if the information is to be exchanged between two platforms, one or both of which may be in motion.

(3) *Broadband ultrasonic transducers for nondestructive testing, medical diagnosis, and materials research.* The fidelity of the transmitted and received signals affects the accuracy of derived parameters from flaws, tissue, and materials. The constant beam characteristics hold for the nearfield as well as for the farfield, thus making the application to highly directive ultrasonic transducers possible.

## I. THEORY

It is well known[7–9] that if the radial velocity on the surface of a rigid sphere of radius $a$ is equal to $U_0 u(\theta) \times e^{-i\omega t}$, where $\omega$ is the angular frequency, then the corresponding acoustic pressure will be

$$p(R,\theta,t) = i\rho c U_0 e^{-i\omega t} \sum_{n=0}^{\infty} A_n P_n(\cos\theta) \frac{h_n(kR)}{h_n'(ka)} , \quad (1)$$

where $R$ and $\theta$ are spherical coordinates, $h_n$ is a spherical Hankel function, $h_n'$ is its derivative, $\rho$ is the density and $c$ the sound speed of the surrounding fluid, and $k = \omega/c$ is the wave number. The quantity $U_0$ is the peak velocity, and $u(\theta)$ is the dimensionless velocity distribution. The quantities $A_n$ are the coefficients in the expansion of $u(\theta)$ in the following series of Legendre polynomials $P_n(\cos\theta)$,

$$u(\theta) = \sum_{n=0}^{\infty} A_n P_n(\cos\theta) , \quad (2)$$

and are defined by

$$A_n = (n + \tfrac{1}{2}) \int_0^{\pi} u(\theta) P_n(\cos\theta) \sin\theta \, d\theta . \quad (3)$$

The farfield pressure, defined as the limit of $p(R,\theta,t)$ when $R \to \infty$, is written as

$$p_{FF}(R,\theta,t) = (\rho c U_0 a \, e^{ik(R-a)}/R) e^{-i\omega t} g(\theta) , \quad (4)$$

where the angular dependence (beam pattern) $g(\theta)$ is given by



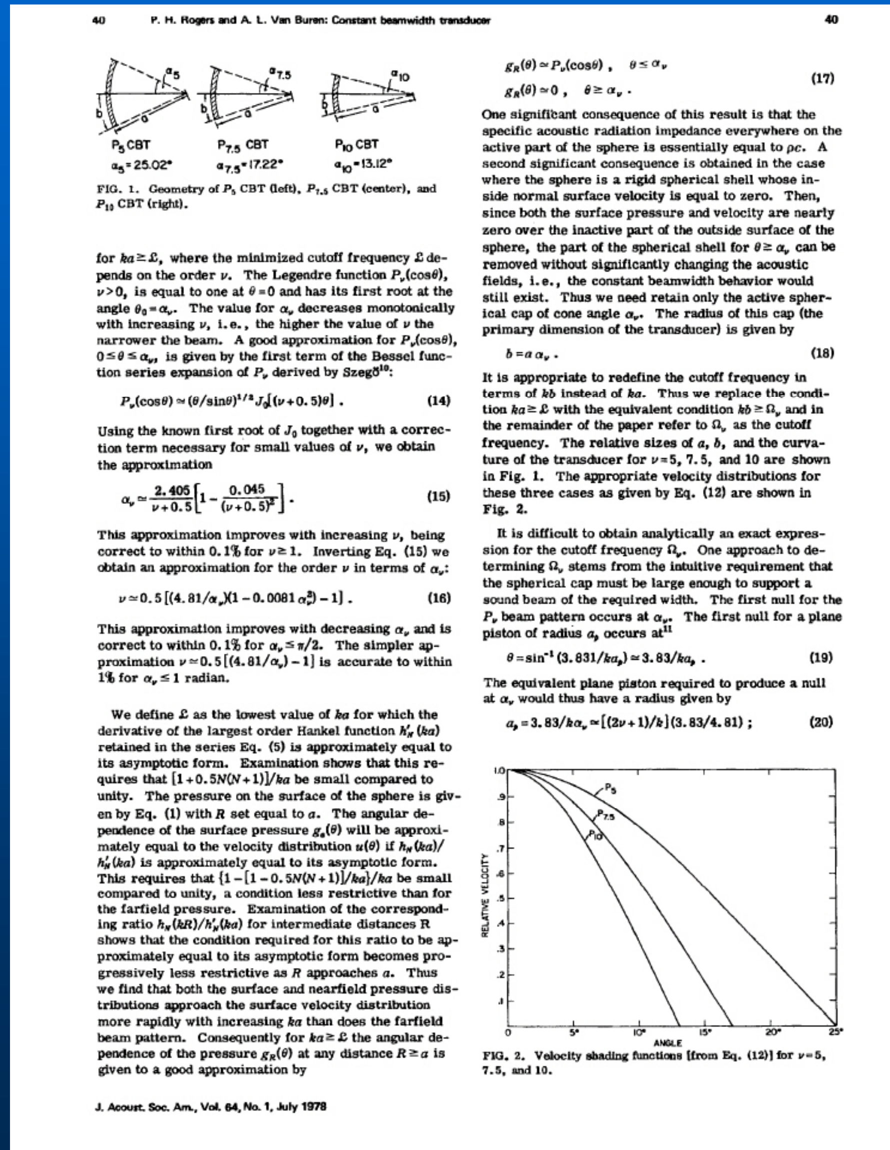






FIG. 1. Geometry of $P_5$ CBT (left), $P_{7.5}$ CBT (center), and $P_{10}$ CBT (right).

for $ka \geq \mathfrak{L}$, where the minimized cutoff frequency $\mathfrak{L}$ depends on the order $\nu$. The Legendre function $P_\nu(\cos\theta)$, $\nu > 0$, is equal to one at $\theta = 0$ and has its first root at the angle $\theta_0 = \alpha_\nu$. The value for $\alpha_\nu$ decreases monotonically with increasing $\nu$, i.e., the higher the value of $\nu$ the narrower the beam. A good approximation for $P_\nu(\cos\theta)$, $0 \leq \theta \leq \alpha_\nu$, is given by the first term of the Bessel function series expansion of $P_\nu$ derived by Szegö[10]:

$$P_\nu(\cos\theta) \simeq (\theta/\sin\theta)^{1/2} J_0[(\nu + 0.5)\theta] . \quad (14)$$

Using the known first root of $J_0$ together with a correction term necessary for small values of $\nu$, we obtain the approximation

$$\alpha_\nu \simeq \frac{2.405}{\nu + 0.5}\left[1 - \frac{0.045}{(\nu + 0.5)^2}\right] . \quad (15)$$

This approximation improves with increasing $\nu$, being correct to within 0.1% for $\nu \geq 1$. Inverting Eq. (15) we obtain an approximation for the order $\nu$ in terms of $\alpha_\nu$:

$$\nu \simeq 0.5[(4.81/\alpha_\nu)(1 - 0.0081\alpha_\nu^2) - 1] . \quad (16)$$

This approximation improves with decreasing $\alpha_\nu$ and is correct to within 0.1% for $\alpha_\nu \leq \pi/2$. The simpler approximation $\nu \simeq 0.5[(4.81/\alpha_\nu) - 1]$ is accurate to within 1% for $\alpha_\nu \leq 1$ radian.

We define $\mathfrak{L}$ as the lowest value of $ka$ for which the derivative of the largest order Hankel function $h_N'(ka)$ retained in the series Eq. (5) is approximately equal to its asymptotic form. Examination shows that this requires that $[1 + 0.5N(N+1)]/ka$ be small compared to unity. The pressure on the surface of the sphere is given by Eq. (1) with $R$ set equal to $a$. The angular dependence of the surface pressure $g_a(\theta)$ will be approximately equal to the velocity distribution $u(\theta)$ if $h_N(ka)/h_N'(ka)$ is approximately equal to its asymptotic form. This requires that $\{1 - [1 - 0.5N(N+1)]/ka\}/ka$ be small compared to unity, a condition less restrictive than for the farfield pressure. Examination of the corresponding ratio $h_N(kR)/h_N'(ka)$ for intermediate distances R shows that the condition required for this ratio to be approximately equal to its asymptotic form becomes progressively less restrictive as $R$ approaches $a$. Thus we find that both the surface and nearfield pressure distributions approach the surface velocity distribution more rapidly with increasing $ka$ than does the farfield beam pattern. Consequently for $ka \geq \mathfrak{L}$ the angular dependence of the pressure $g_R(\theta)$ at any distance $R \geq a$ is given to a good approximation by

$$g_R(\theta) \simeq P_\nu(\cos\theta) , \quad \theta \leq \alpha_\nu$$
$$g_R(\theta) \simeq 0 , \quad \theta \geq \alpha_\nu . \quad (17)$$

One significant consequence of this result is that the specific acoustic radiation impedance everywhere on the active part of the sphere is essentially equal to $\rho c$. A second significant consequence is obtained in the case where the sphere is a rigid spherical shell whose inside normal surface velocity is equal to zero. Then, since both the surface pressure and velocity are nearly zero over the inactive part of the outside surface of the sphere, the part of the spherical shell for $\theta \geq \alpha_\nu$ can be removed without significantly changing the acoustic fields, i.e., the constant beamwidth behavior would still exist. Thus we need retain only the active spherical cap of cone angle $\alpha_\nu$. The radius of this cap (the primary dimension of the transducer) is given by

$$b = a\,\alpha_\nu . \quad (18)$$

It is appropriate to redefine the cutoff frequency in terms of $kb$ instead of $ka$. Thus we replace the condition $ka \geq \mathfrak{L}$ with the equivalent condition $kb \geq \Omega_\nu$ and in the remainder of the paper refer to $\Omega_\nu$ as the cutoff frequency. The relative sizes of $a$, $b$, and the curvature of the transducer for $\nu = 5$, 7.5, and 10 are shown in Fig. 1. The appropriate velocity distributions for these three cases as given by Eq. (12) are shown in Fig. 2.

It is difficult to obtain analytically an exact expression for the cutoff frequency $\Omega_\nu$. One approach to determining $\Omega_\nu$ stems from the intuitive requirement that the spherical cap must be large enough to support a sound beam of the required width. The first null for the $P_\nu$ beam pattern occurs at $\alpha_\nu$. The first null for a plane piston of radius $a_p$ occurs at[11]

$$\theta = \sin^{-1}(3.831/ka_p) \simeq 3.83/ka_p . \quad (19)$$

The equivalent plane piston required to produce a null at $\alpha_\nu$ would thus have a radius given by

$$a_p = 3.83/k\alpha_\nu \simeq [(2\nu + 1)/k](3.83/4.81) ; \quad (20)$$



FIG. 2. Velocity shading functions [from Eq. (12)] for $\nu = 5$, 7.5, and 10.

# Highlighted Text

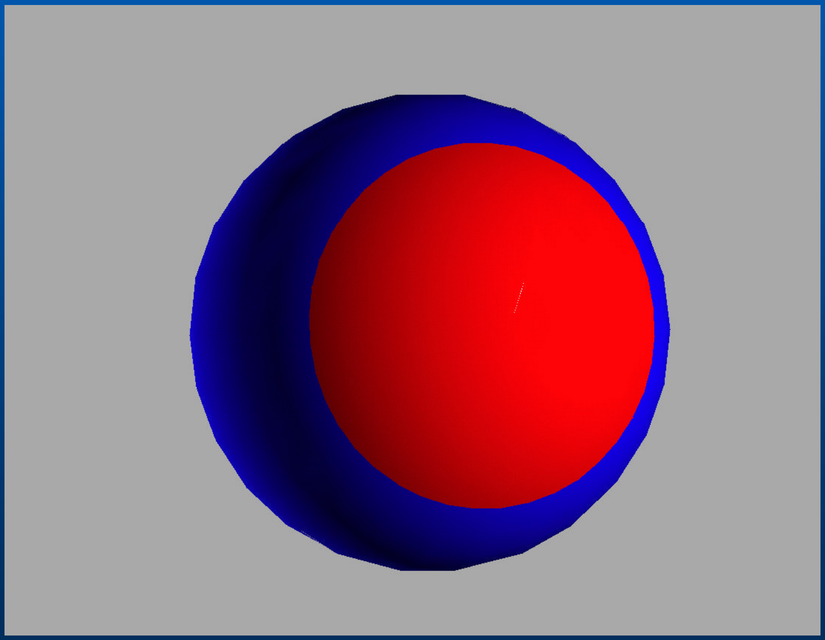
- “With such a ‘constant beamwidth transducer’ (CBT) the spectral content of the acoustic signal would be independent of bearing.”

# Spherical-Cap CBT Transducers

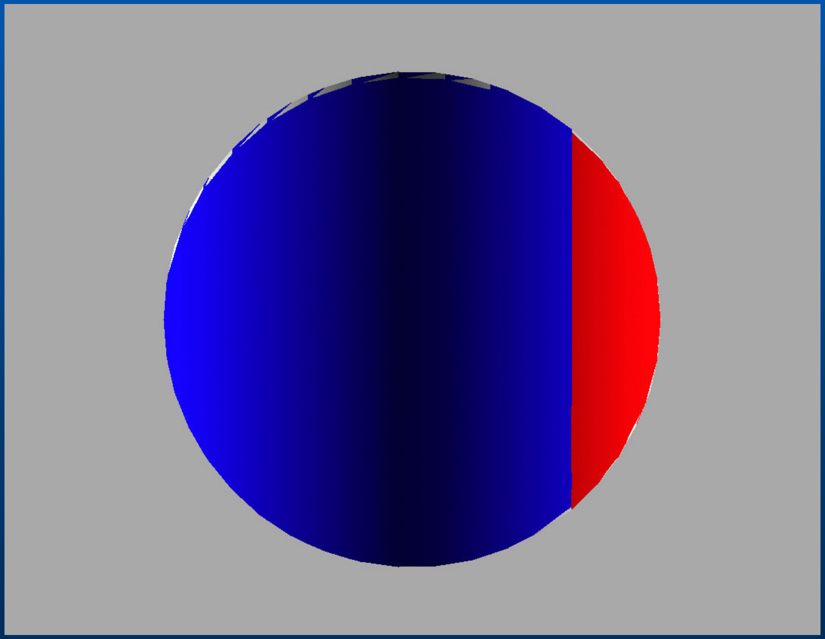
## Overview

### 100° Circular Spherical Cap

Oblique View



Side View

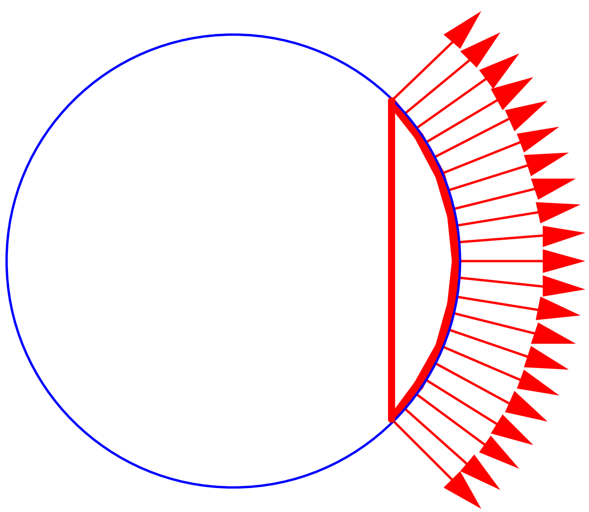


# Spherical-Cap CBT Transducers

## Overview Cont.:
## Legendre Shading of Surface Pressure

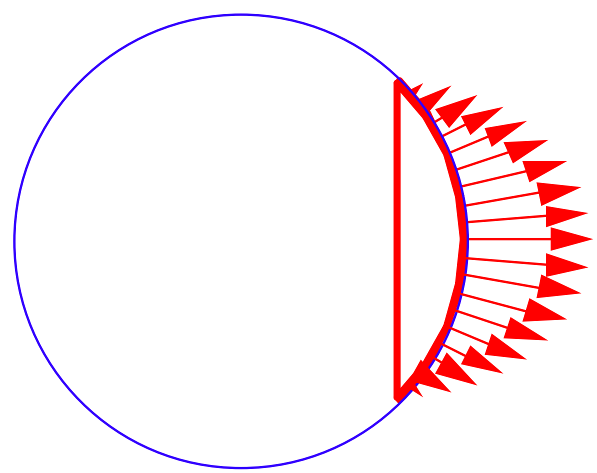
### No Shading

$$p(\theta)=\begin{cases}1 & \text{for} \quad \theta \le \theta_0 \\ 0 & \text{for} \quad \theta > \theta_0\end{cases}$$



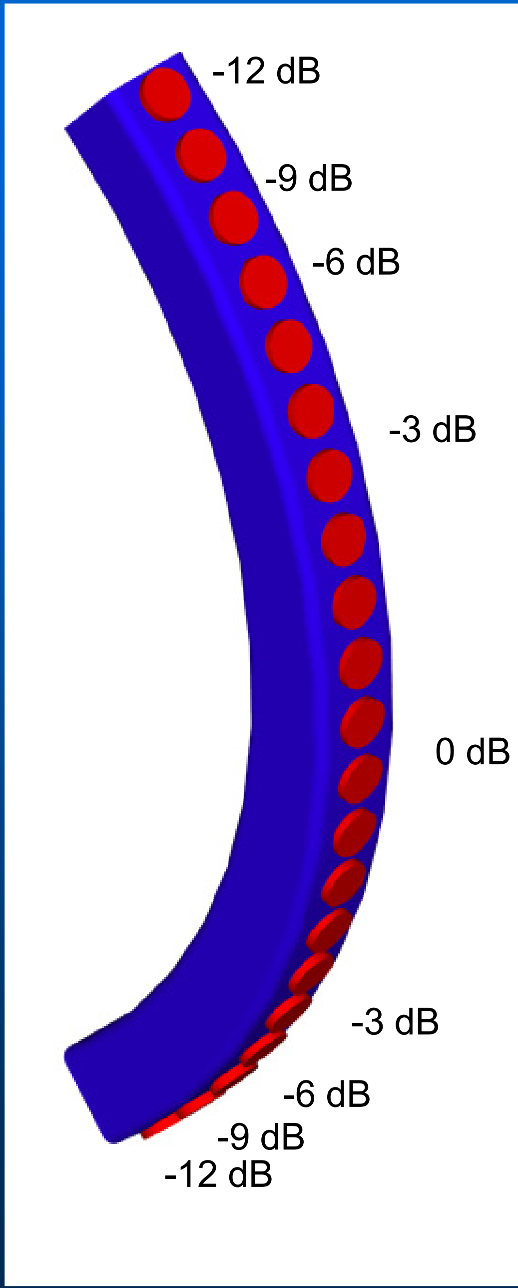
### With Shading

$$p(\theta)=\begin{cases}P_v(\cos\theta) & \text{for} \quad \theta \le \theta_0 \\ 0 & \text{for} \quad \theta > \theta_0\end{cases}$$



# Spherical-Cap CBT Transducers Overview Cont.: Observations

- Provides extremely uniform polars above a certain frequency which are independent of distance
- Beamwidth = 0.64 x Cap Angle
- Surface pressure distribution, nearfield pressure pattern, and farfield pressure pattern are all essentially the same! No nearfield!
- Don’t need the rest of the sphere!

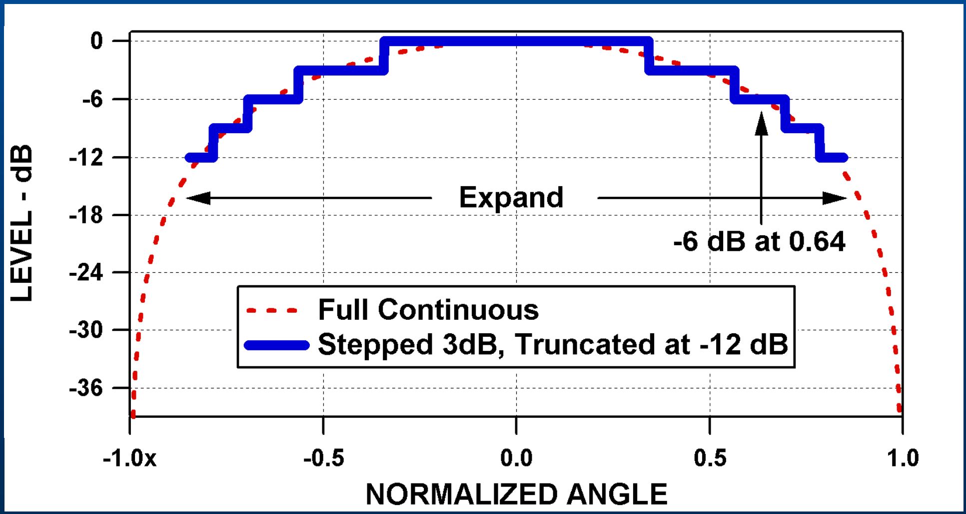



# Applied to Circular-Arc Line Arrays

## CBT Curved Line Source (Circular Wedge)

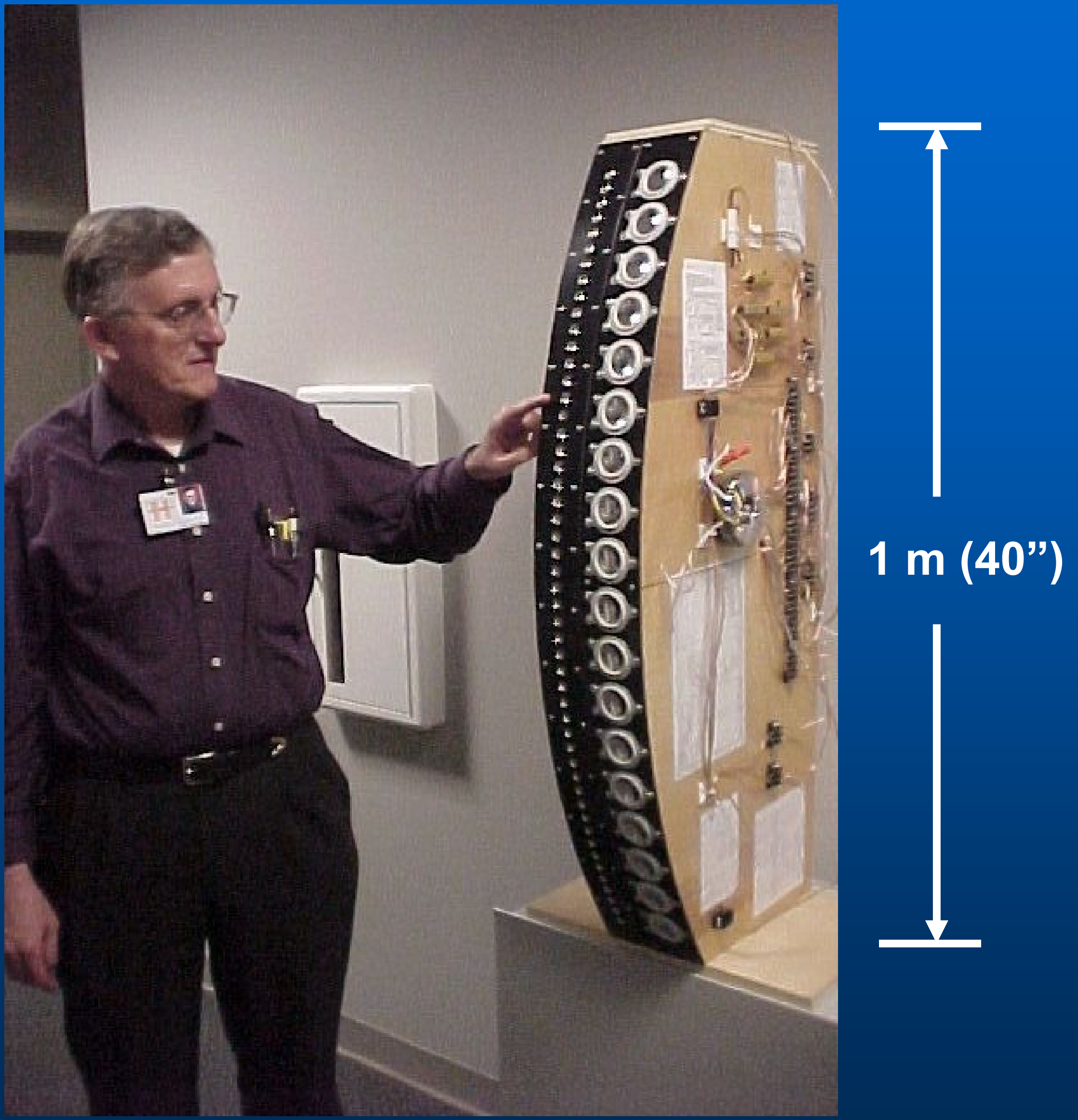
### Truncated and Stepped Legendre Shading

(Power Loss = 2.3 dB)



# First CBT Line-Array Prototype of 2003

(Passive Shading)



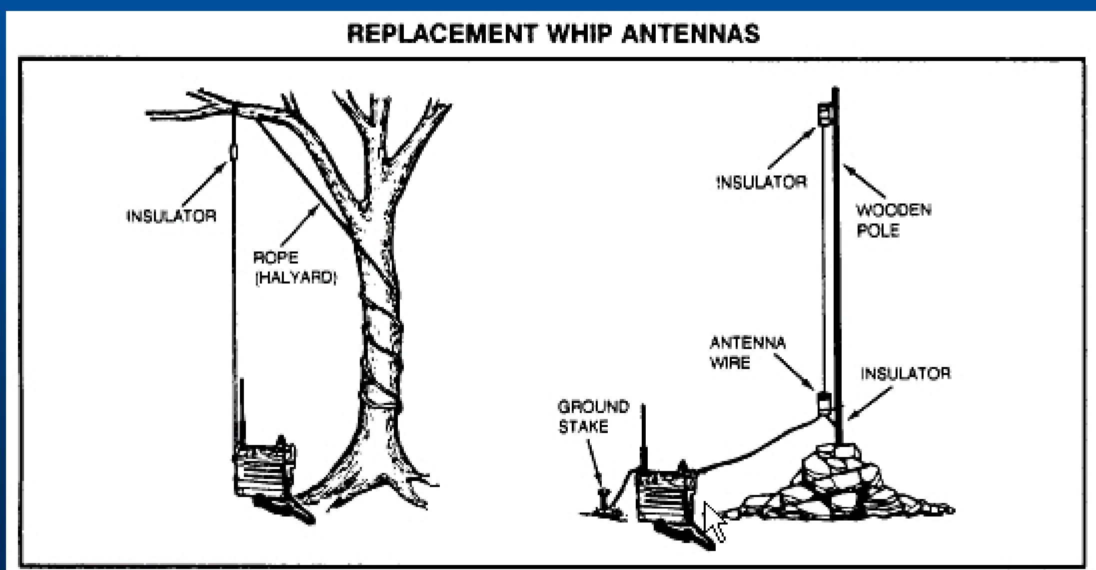


- 18 Ea 57mm (2.25") drivers
- 50 Ea 19mm (0.75") drivers.

# Ground-Plane RF Antennas


## Common Whip Antennas

One half of a half-wave dipole mounted over an electromagnetic ground plane that supplies the missing half.




# Ground-Plane Techniques Have Never Been Applied to Loudspeakers Before!

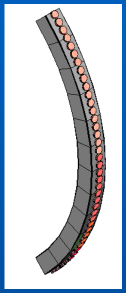
# Personalized Amplification System™


Photos courtesy Bose Corp.
(www.bose.com/musicians)

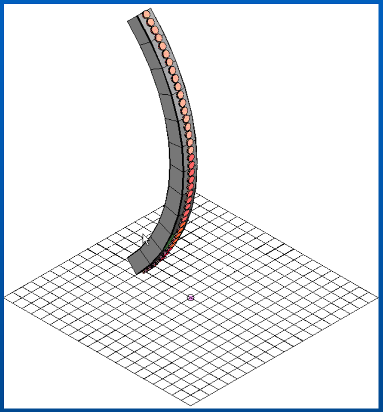

The final protoytpe of the Cylindrical Radiator loudspeaker is shown (with inventor Clifford Henricksen).

# Steps to Create a Ground-Plane Array from a Free-Standing CBT Array

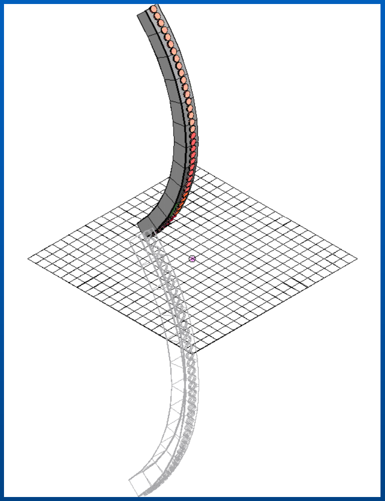
1. Free-standing array



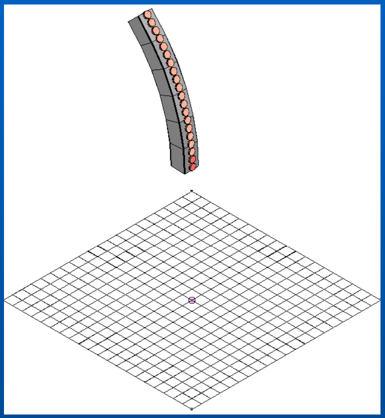
2. Free-standing array over ground plane



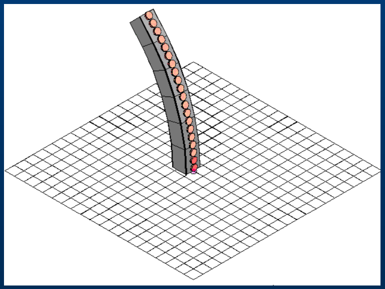
3. Free-standing array over ground plane with reflection



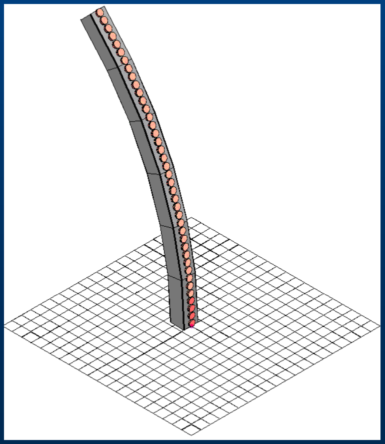
4. Chop off bottom half of array



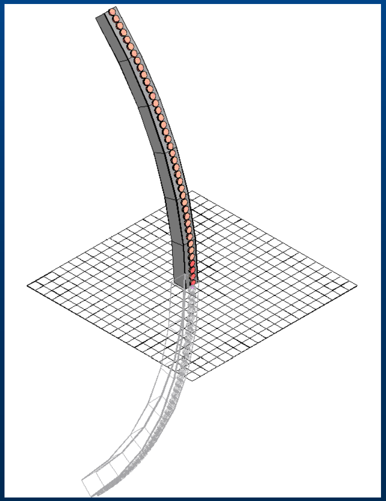
5. Lower to ground plane



6. Now double its size



7. Array is now effectively double the size of the original



Keele, Oct. 7, 2005

Ground-Plane CBT Line Arrays

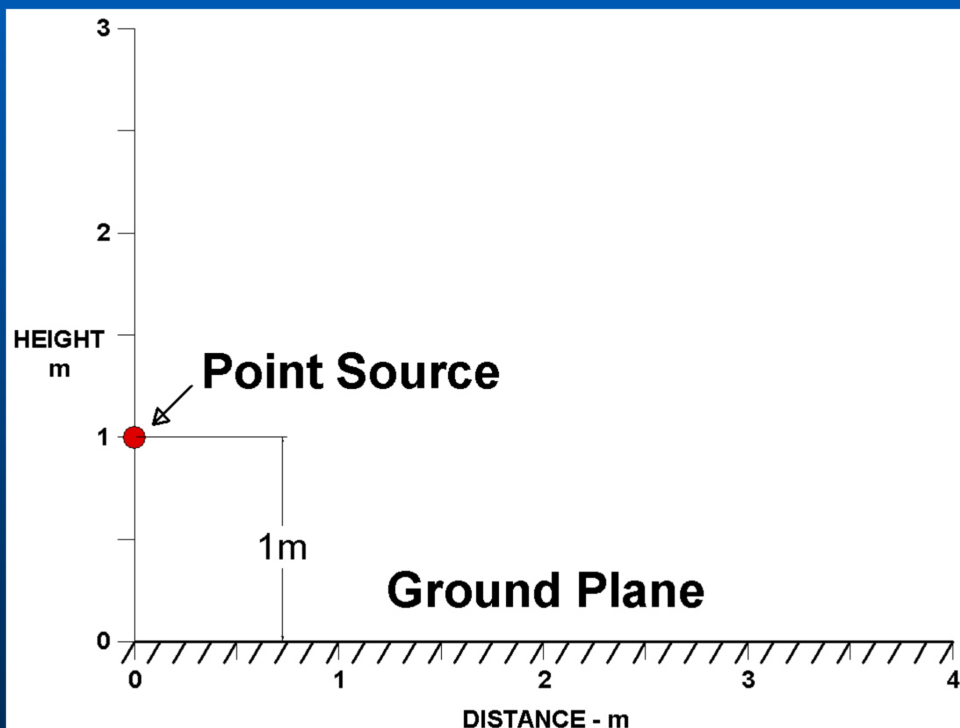
# Vertical-Plane Sound-Field Simulations and Movies

- Four Configurations Were Analyzed Over a Ground Plane (all point-source arrays):
  - Single point source
  - Typical three-way system with 2nd-order Linqwitz-Riley crossovers at 200 Hz and 2 kHz
  - Straight-line source, no shading
  - Circular-arc curved-line source with Legendre shading

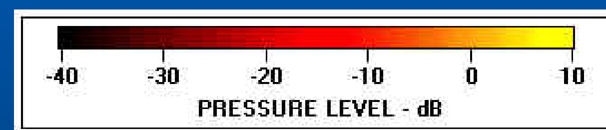
- Additional Configurations Analyzed in Paper but not Shown Here
  - Straight-line source, with Hann shading
  - Circular-arc curved-line source, no shading
  - Delay-curved straight-line source with/without Legendre shading

# Single-Point Source Over a Ground Plane:

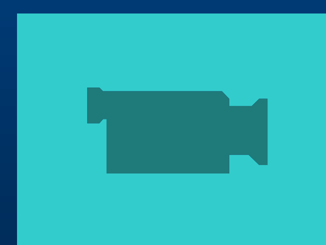
(Non-Reflective vs. Reflective)




Sound-Field Movie
False-Color Scale

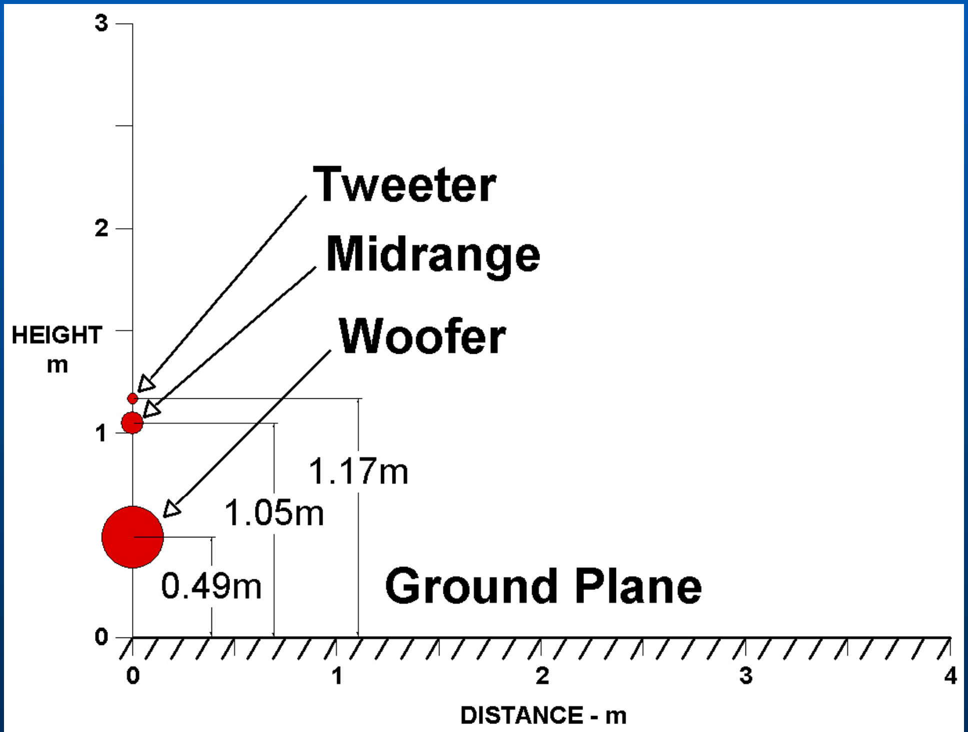



Constant-Pressure
Contours (Isobar)
Every 3 dB



# Three-Way Floor-Standing System Over a Ground Plane (point sources):

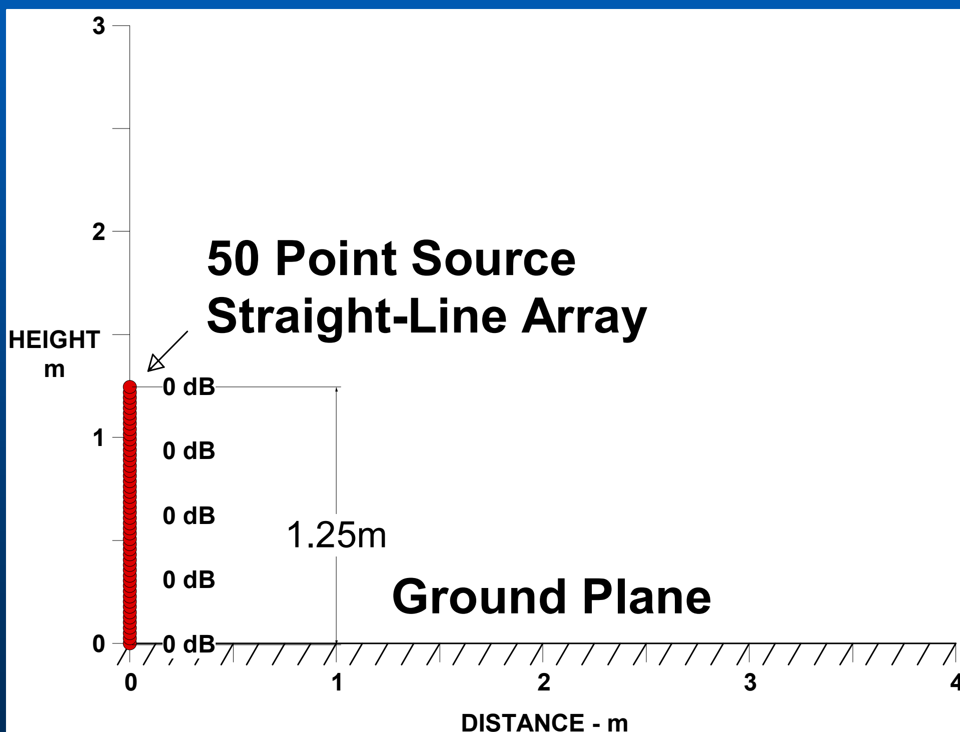
(Non-Reflective vs. Reflective)



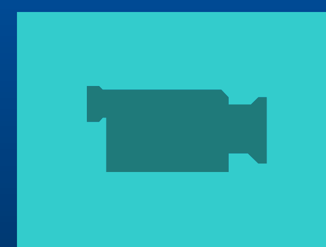

Linqwitz-Riley crossovers at 200 Hz and 2 kHz.

# Un-Shaded Straight-Line Source over a Ground Plane:

(Non-Reflective vs. Reflective)

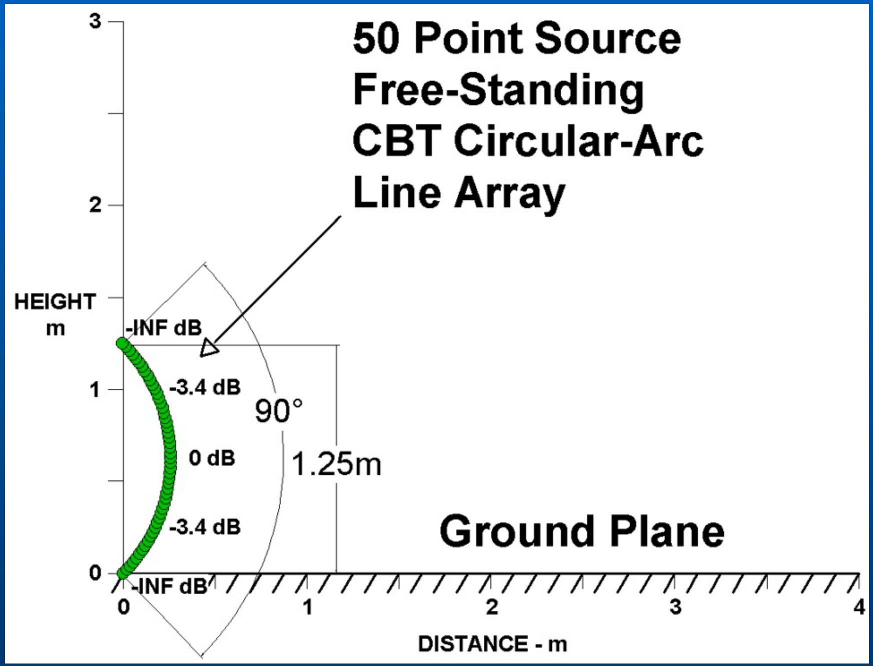


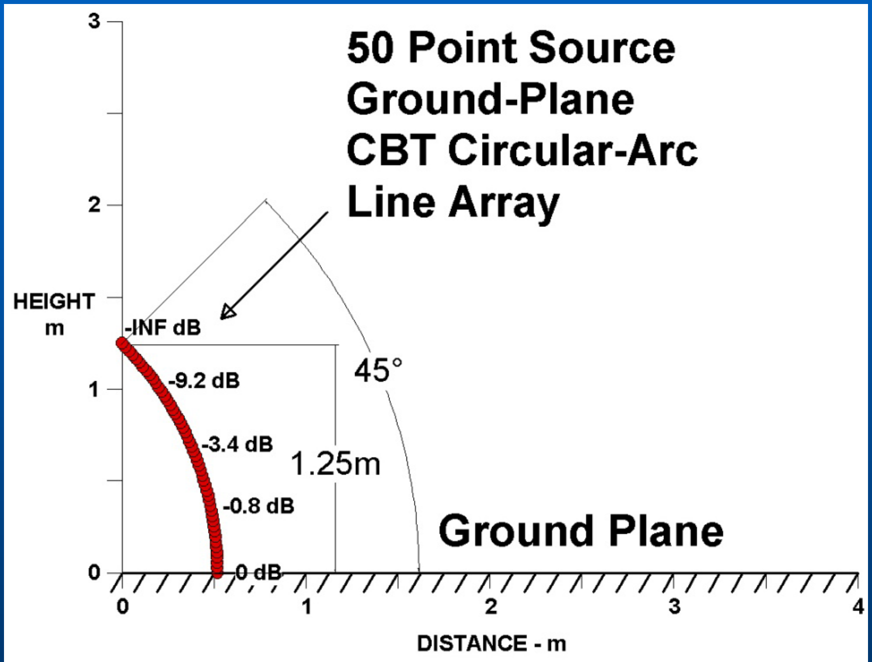

Watch for grating lobes!

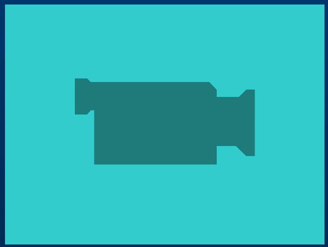


# Legendre-Shaded Circular-Arc Line Sources over a Ground Plane:

(Both Reflective!!)



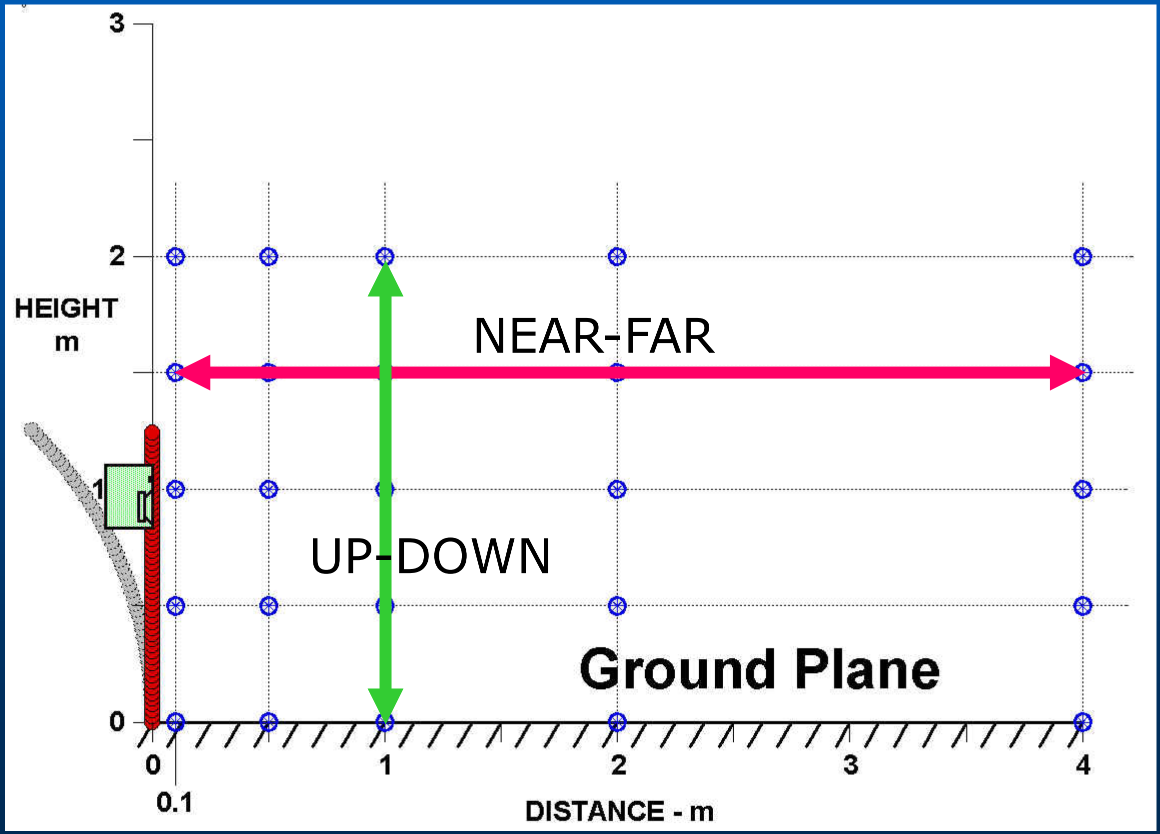



# Measurements: Frequency Response Over the Ground Plane

- Measured three different systems:
  - A conventional two-way compact powered monitor
  - Experimental straight-line array, no shading.
  - Experimental ground-plane circular-arc curved line array, Legendre shading (a CBT array).
- Measurement technique:
  - All measurements were taken with a “Farina” log sine-sweep technique covering the frequency range of 100 to 20 kHz.
  - The resultant impulse response was windowed with a 50 ms Hann window centered over the signal’s first arrival.
  - The windowed time response was then converted to the frequency domain and then sampled at 1/100th-decade intervals.
  - The resultant frequency response was either displayed as is or smoothed with a 1/10th-octave five-point binomial Gaussian filter.

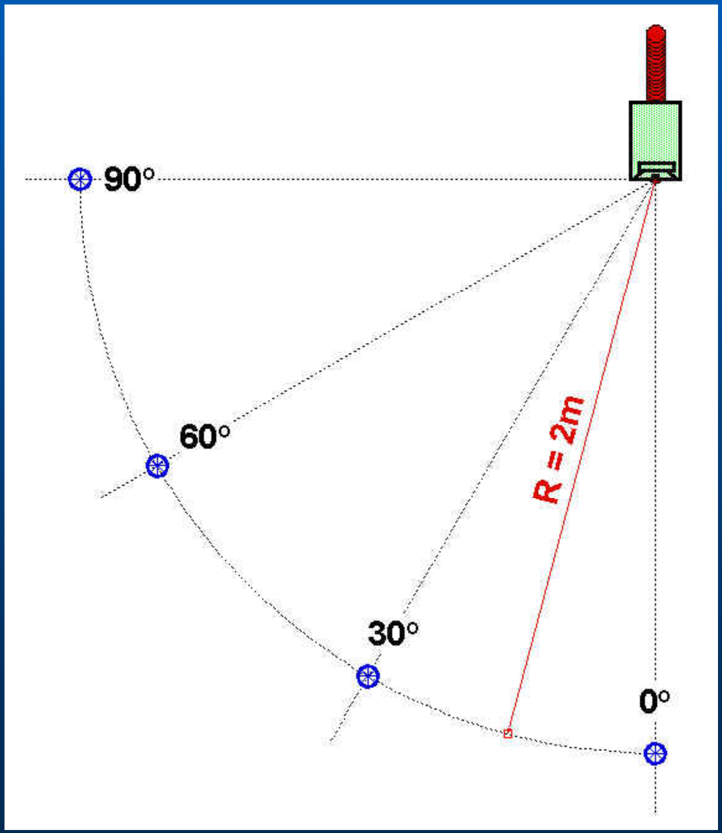
# Measurements: Frequency Response Over the Ground Plane

- Measured frequency response magnitude at 25 grid points in the vertical plane of a 3 m high by 4 m deep region over a reflective ground plane.



# Measurements: Frequency Response Over the Ground Plane

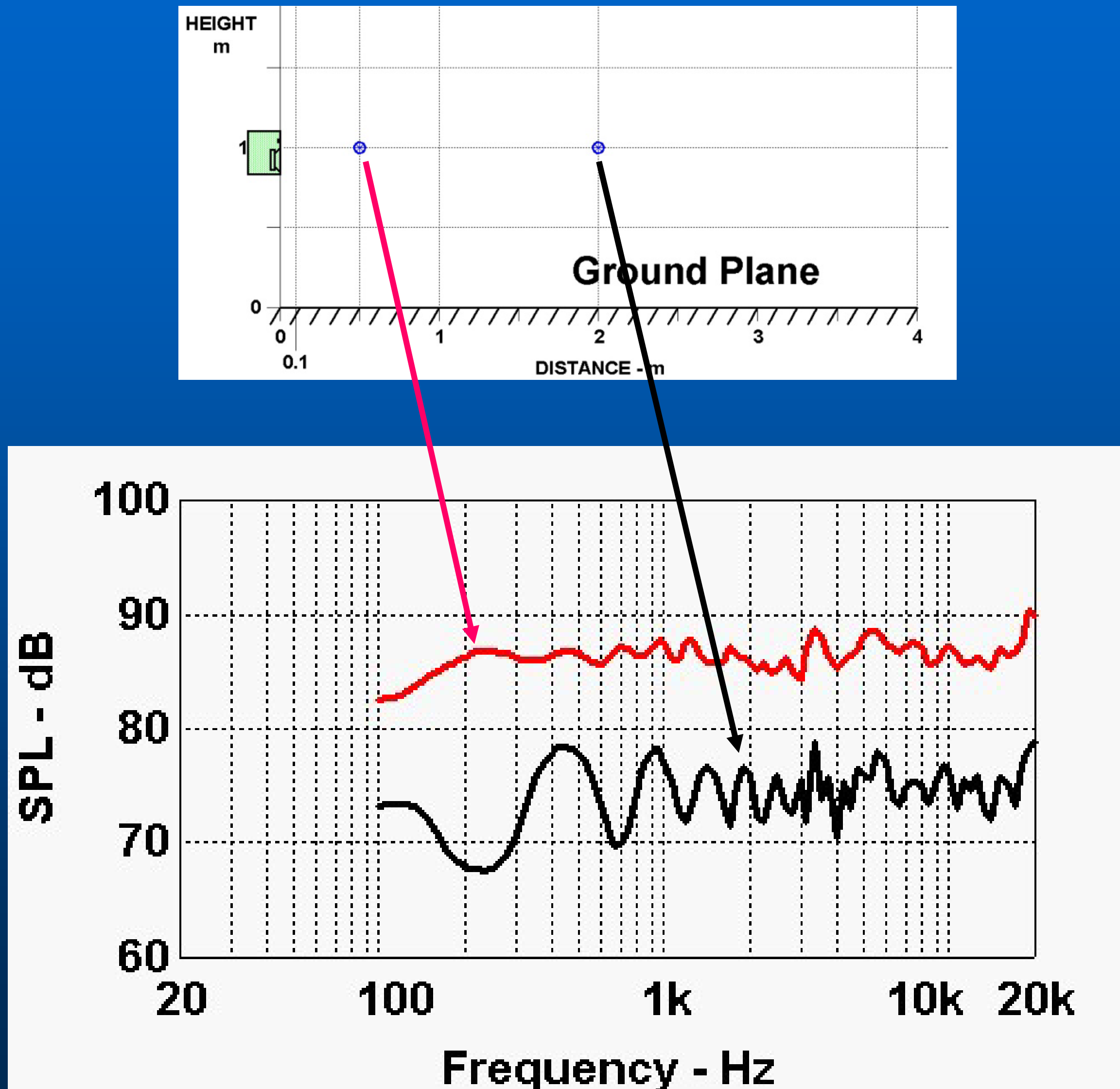
- Measured off-axis frequency response magnitude at 4 points in the horizontal plane at two meters over a reflective ground plane at a height of 0 m and 1 m.



# Conventional Two-Way Compact Powered Monitor


A JBL model LSR 25P mounted on a stand. The height of the monitor was raised so that its axis was 1 m high (a point midway between the tweeter and woofer).

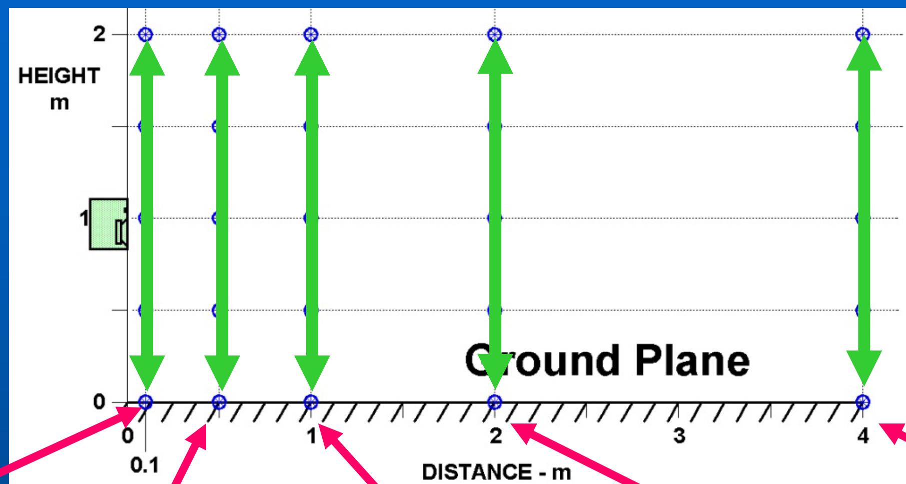
# On-Axis Response (Unsmoothed)

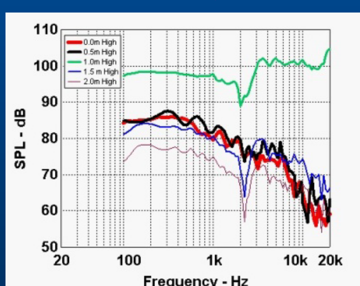


# Response vs. Height
## (at Different Distances)

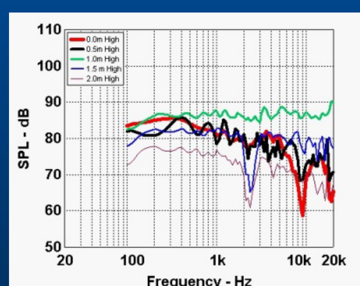
Color Code

- 0.0 m High
- 0.5 m High
- 1.0 m High
- 1.5 m High
- 2.0 m High

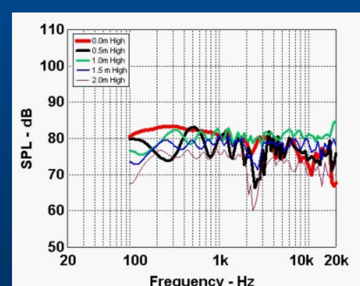


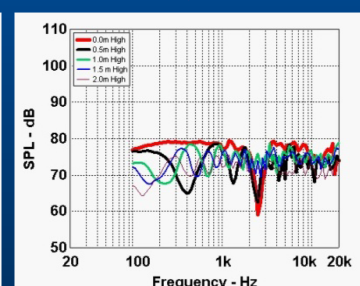

0.1 m



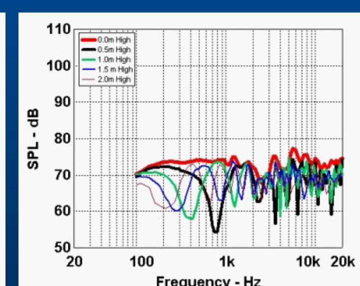

0.5 m




1 m




2 m



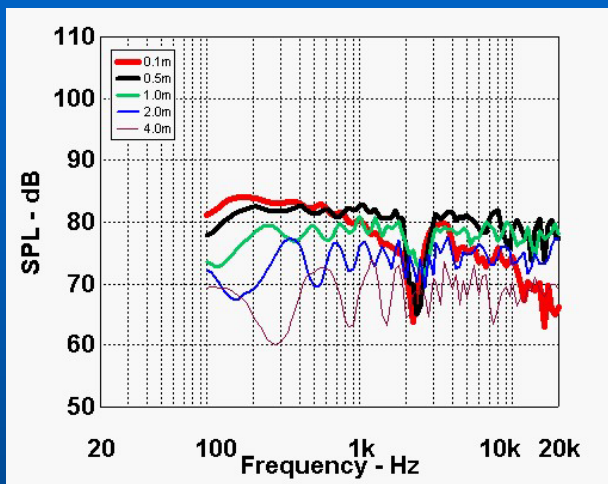

4 m

# Response vs. Distance

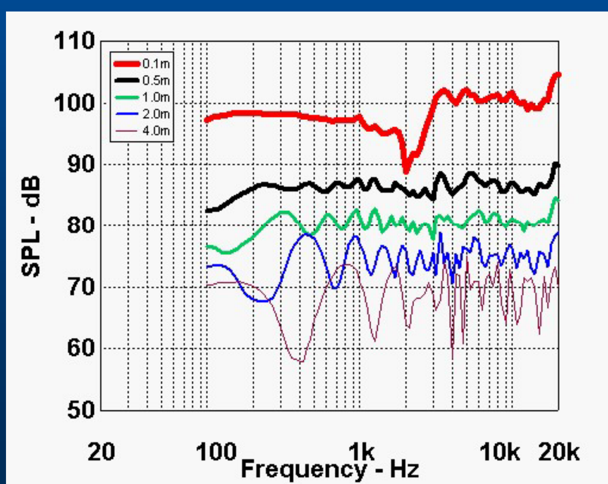
## (at Two Different Heights)

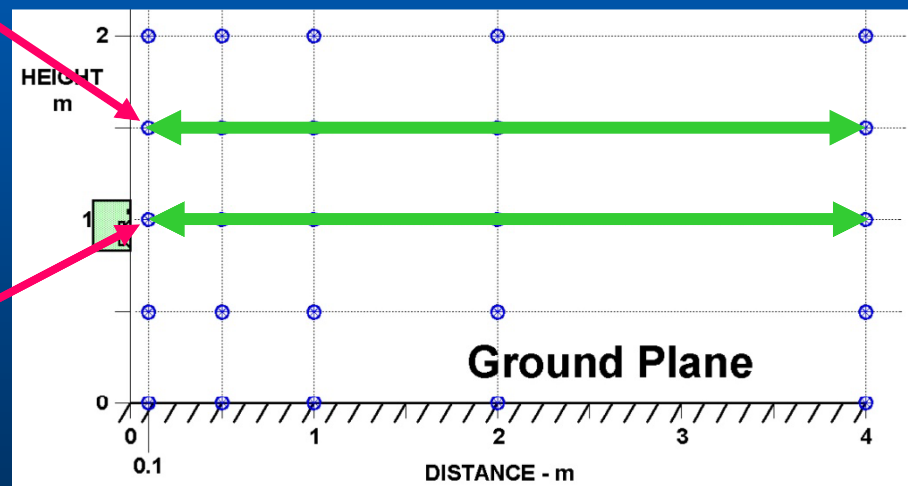
Color Code


1.5 m High




1 m High

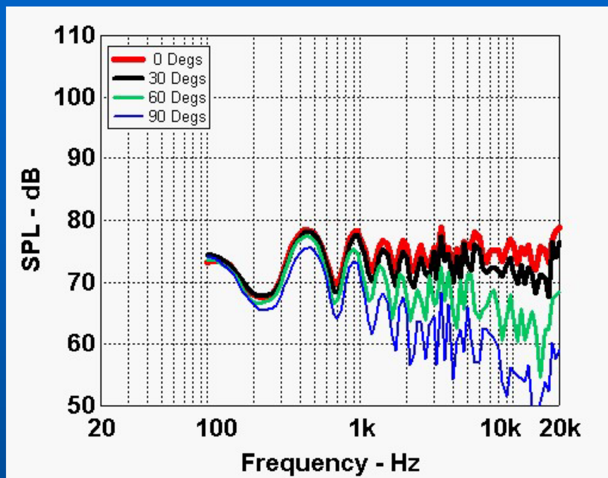


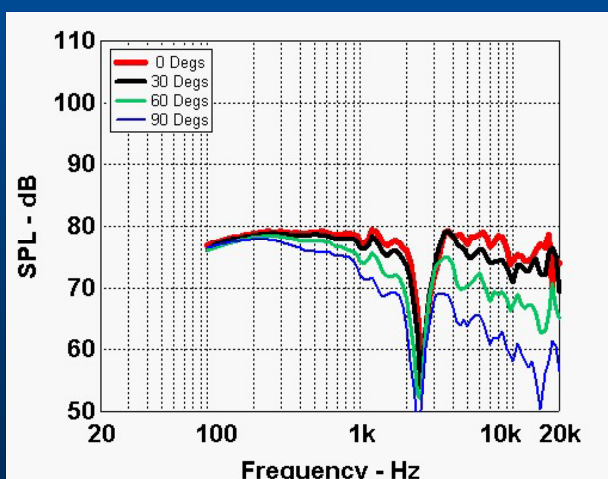
# Response vs. Angle

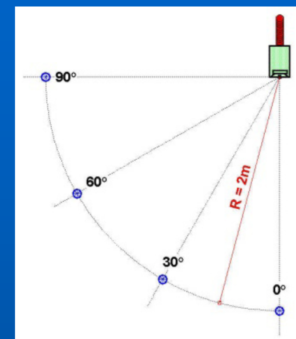
## (at Two Different Heights)

1 m High

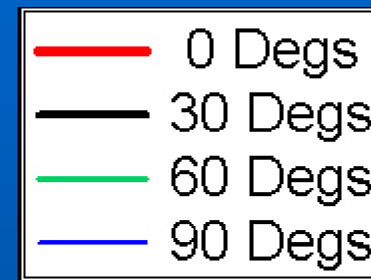


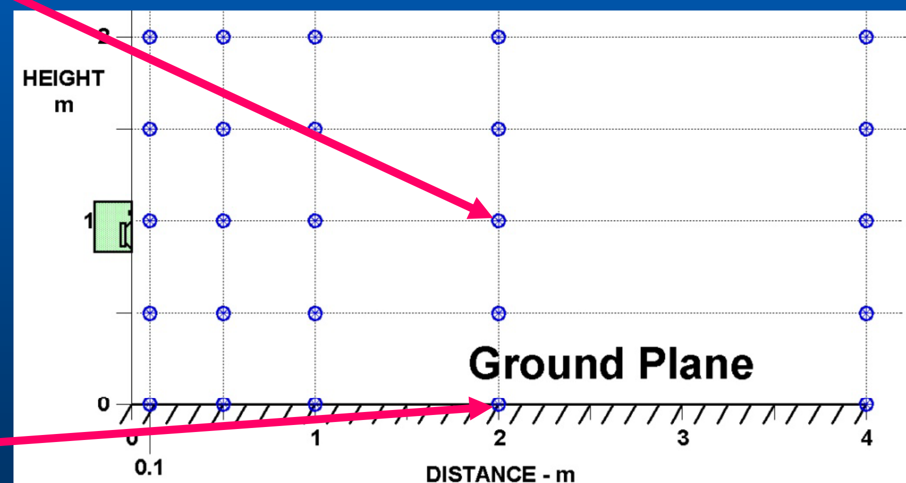

0 m High



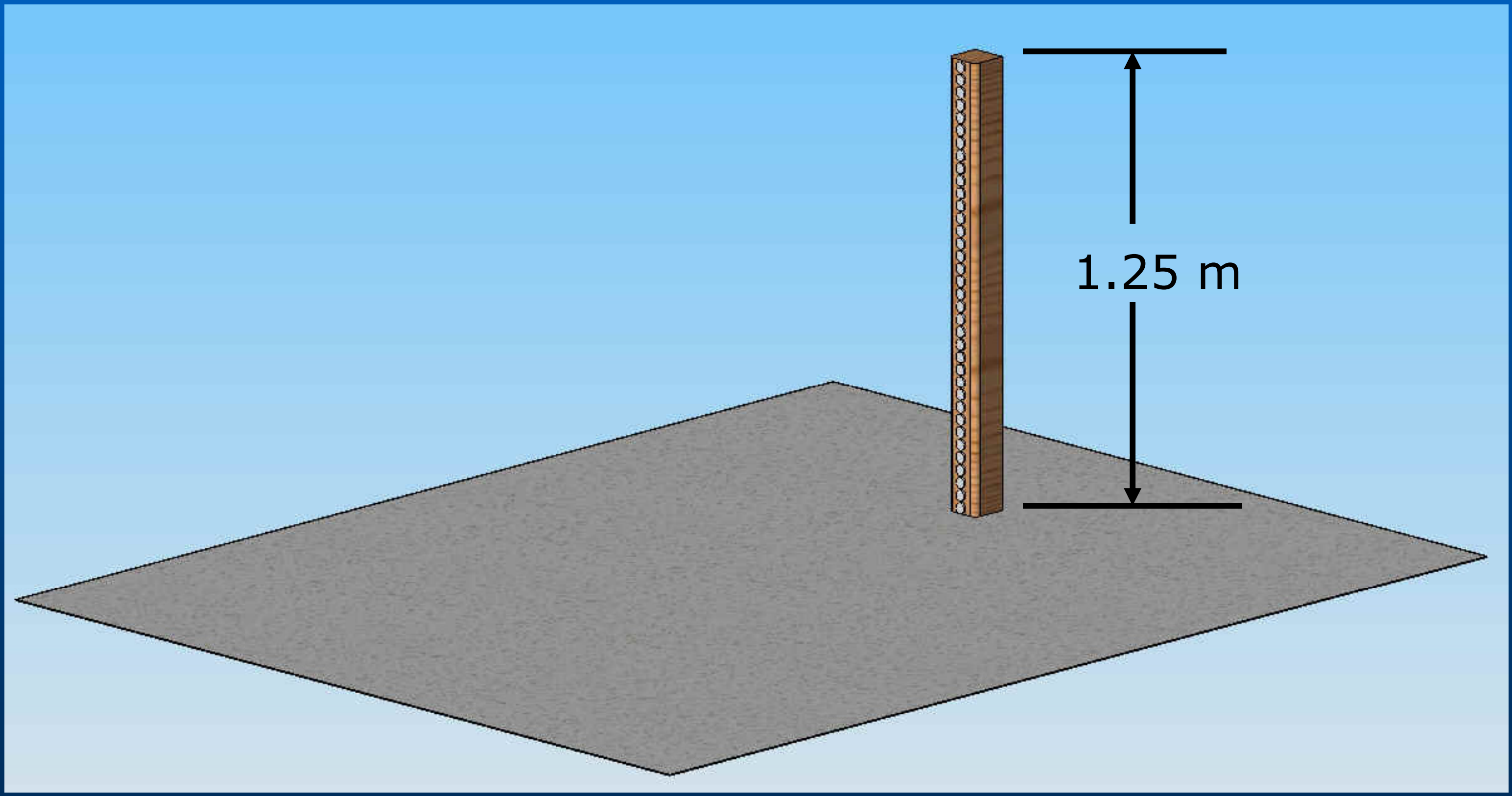

Color Code



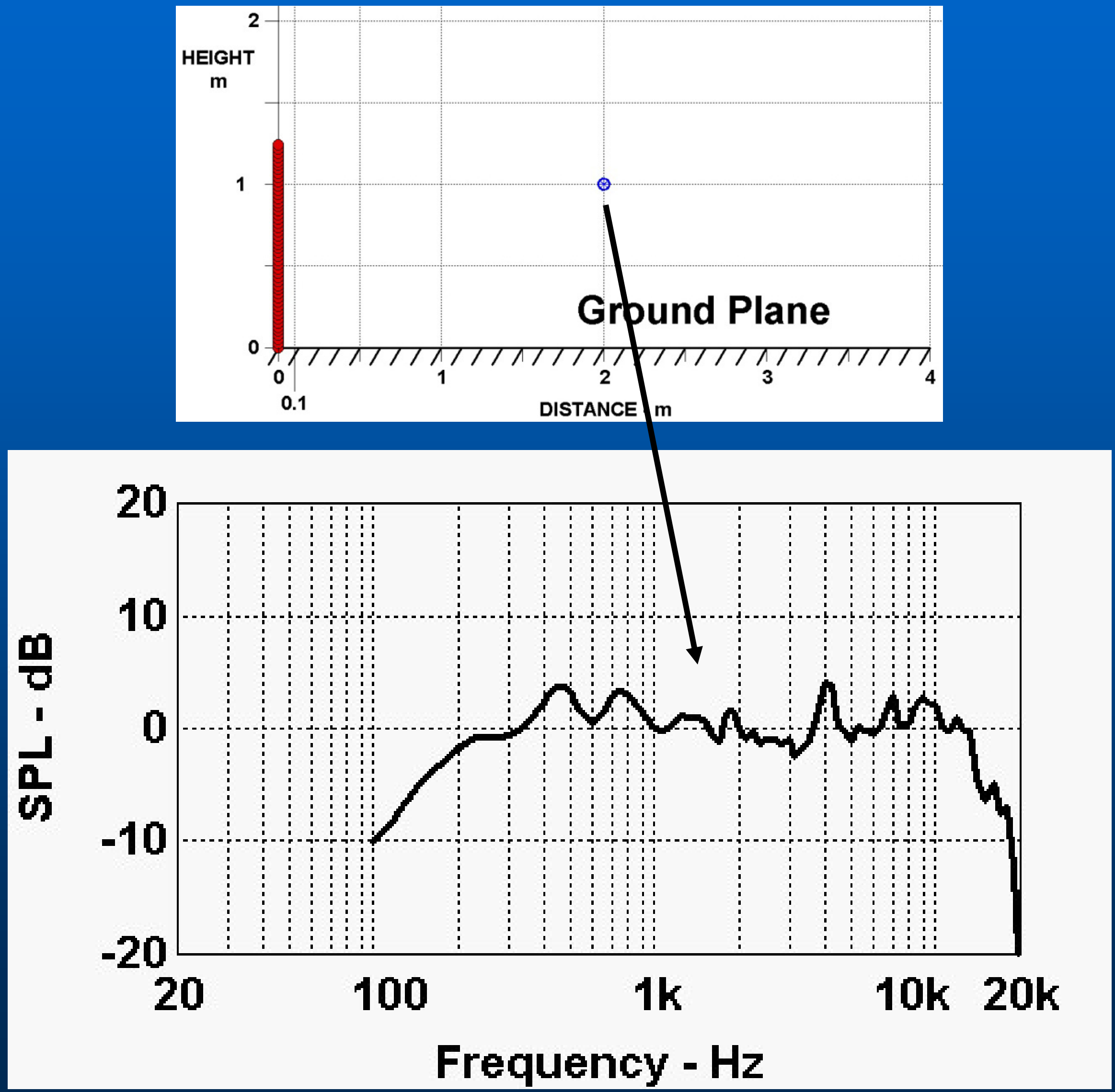



# Experimental Straight-Line Array

The array is 1.25 m high and contains 36 miniature 30 mm wide-range drivers. All drivers were driven with equal in-phase levels.



# On-Axis Response (Unsmoothed)



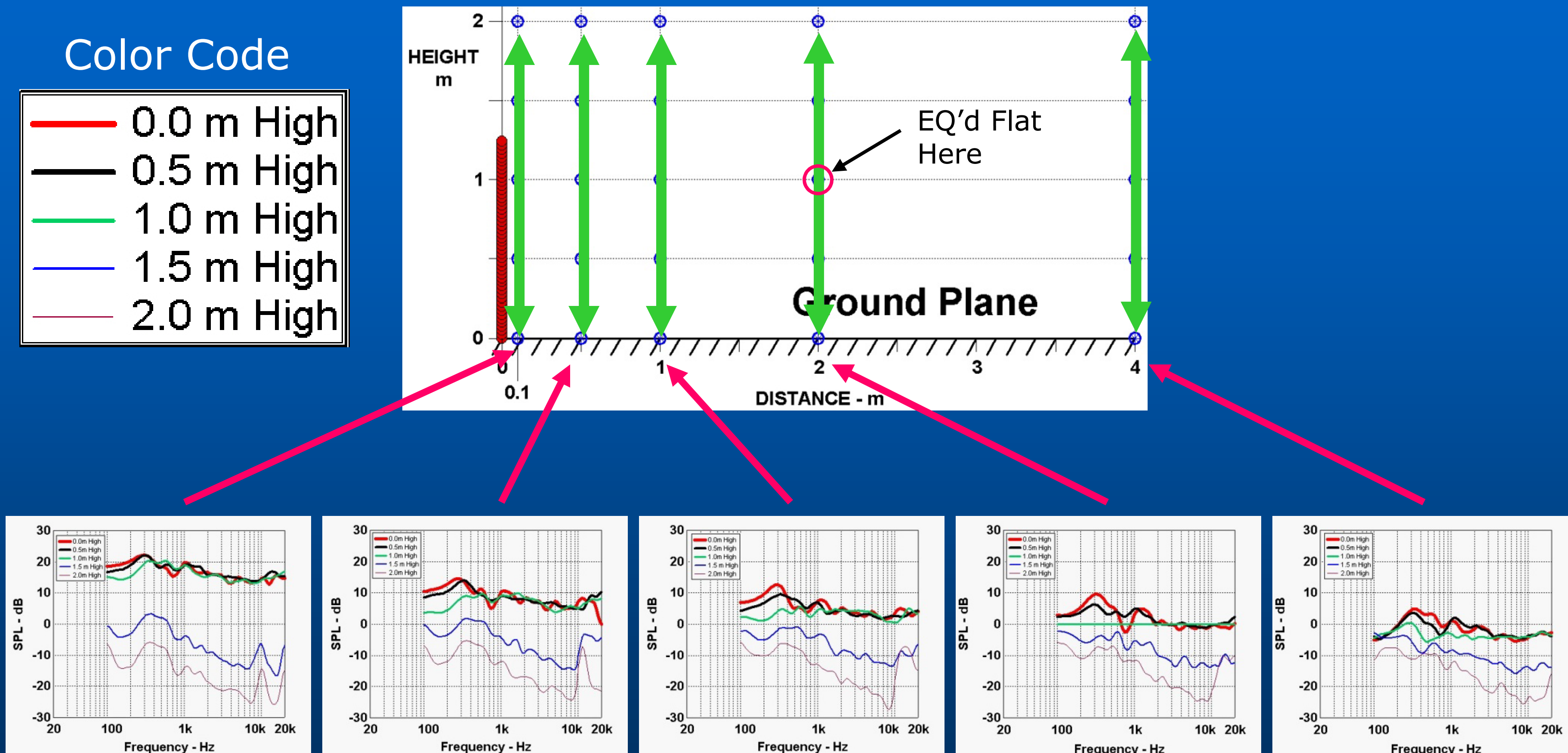

(Normalized to 0 dB at 1 kHz)

# Response vs. Height

## (at Different Distances)

Color Code

- 0.0 m High
- 0.5 m High
- 1.0 m High
- 1.5 m High
- 2.0 m High

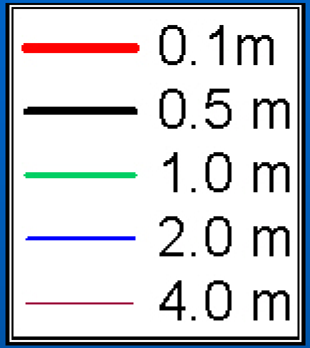



0.1 m 0.5 m 1 m 2 m 4 m

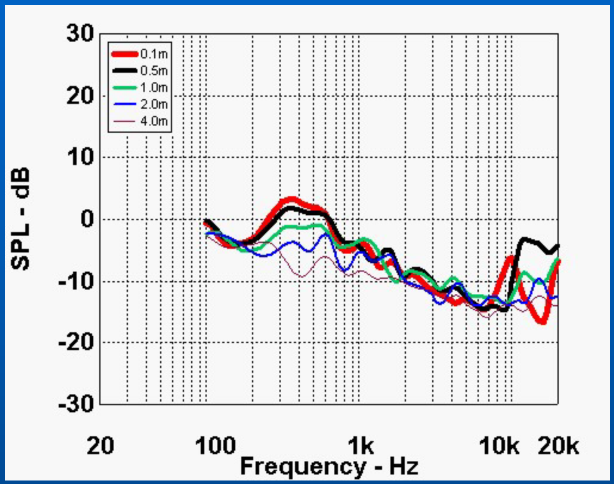
# Response vs. Distance

(at Two Different Heights)

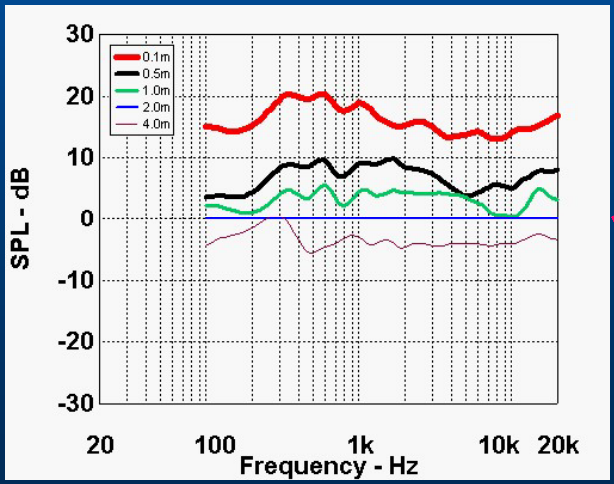
Color Code

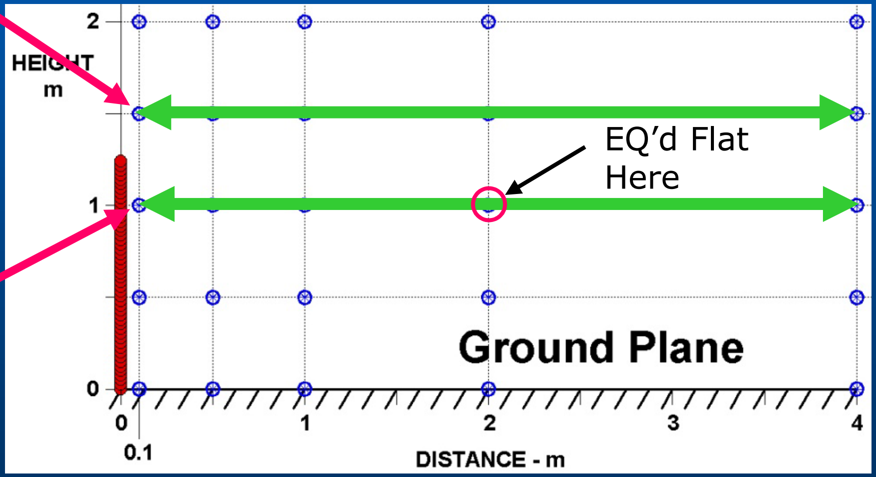



1.5 m High




1 m High



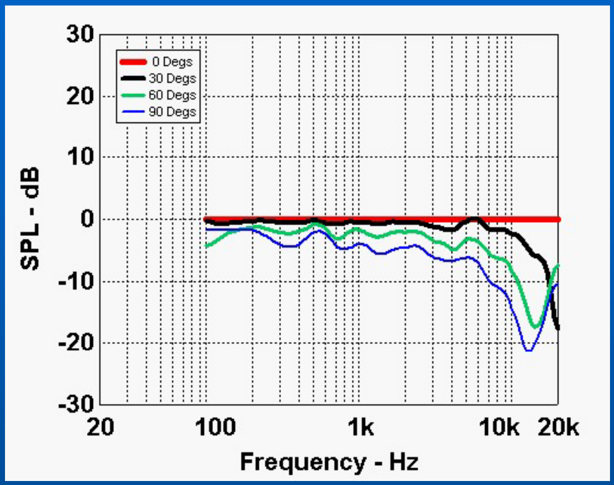
# Response vs. Angle

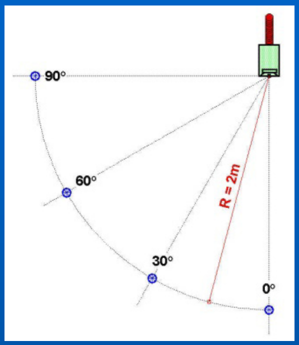
## (at Two Different Heights)

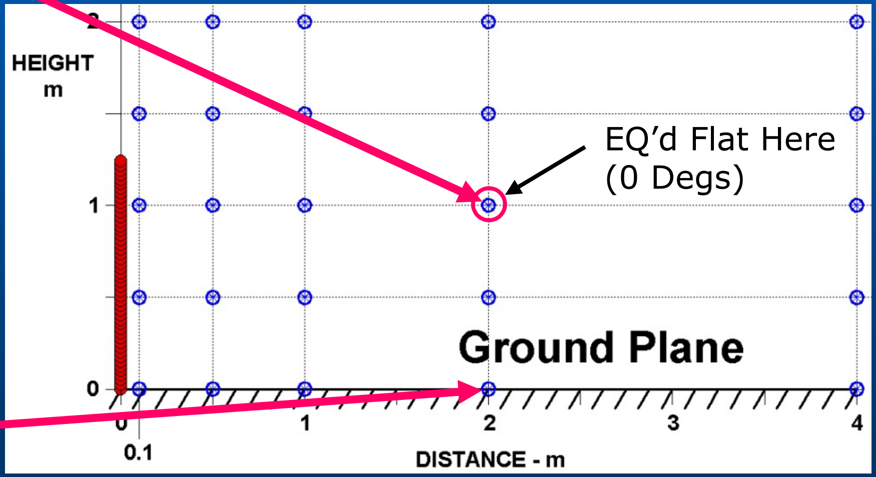
Color Code

- 0 Degs
- 30 Degs
- 60 Degs
- 90 Degs

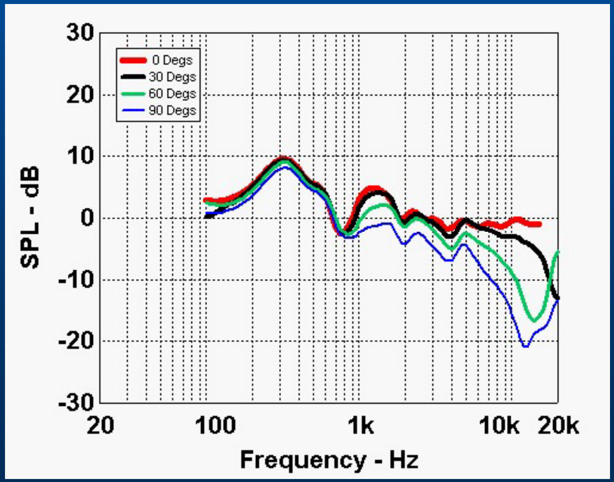
1 m High



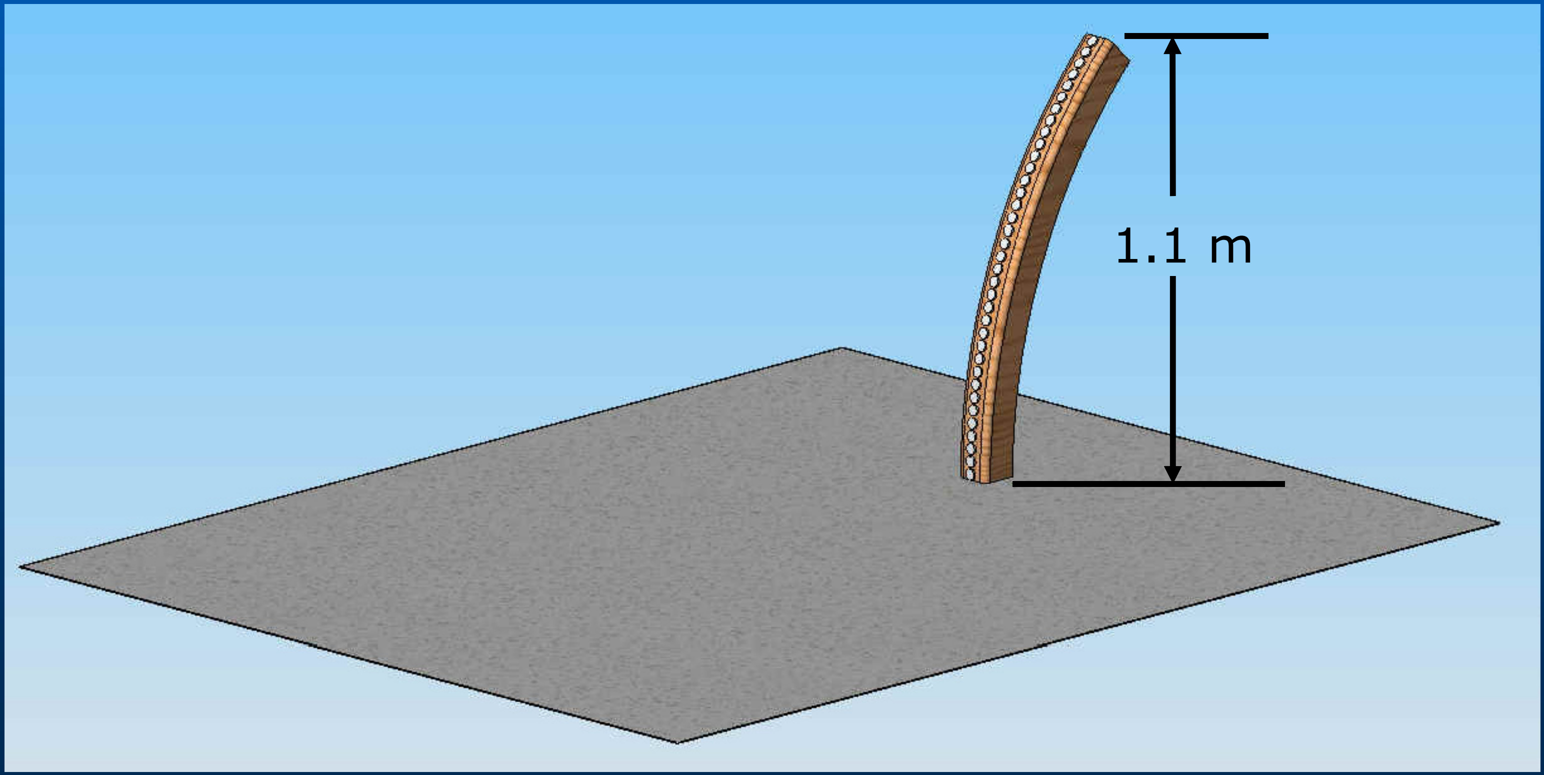




0 m High

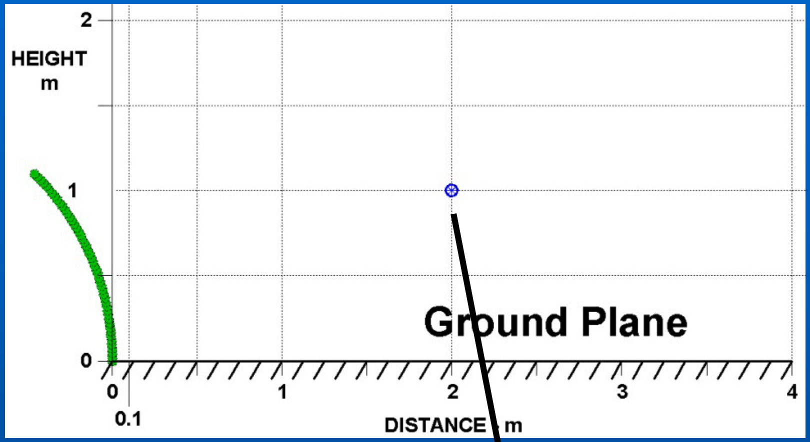


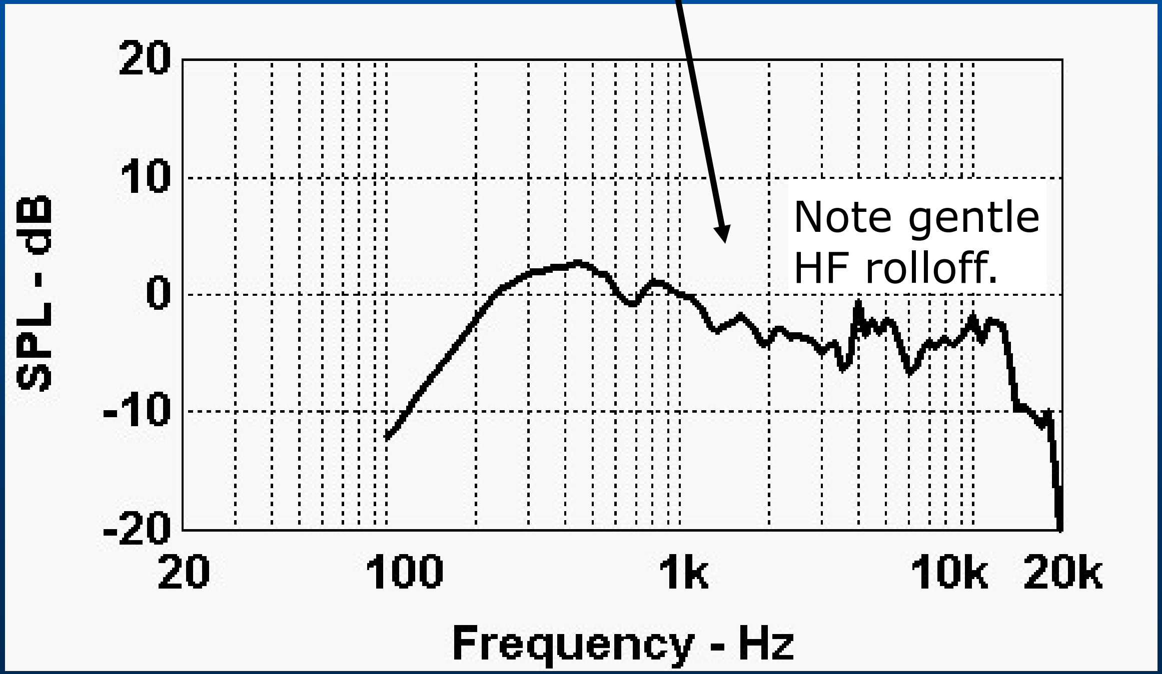
# Experimental Circular-Arc Curved-Line CBT Array

The 45º circular-arc array is 1.1 m high and contains 36 miniature 30 mm wide-range drivers. Legendre shading (frequency independent). Driven by three 12-channel automotive amplifiers.



# On-Axis Response (Unsmoothed)



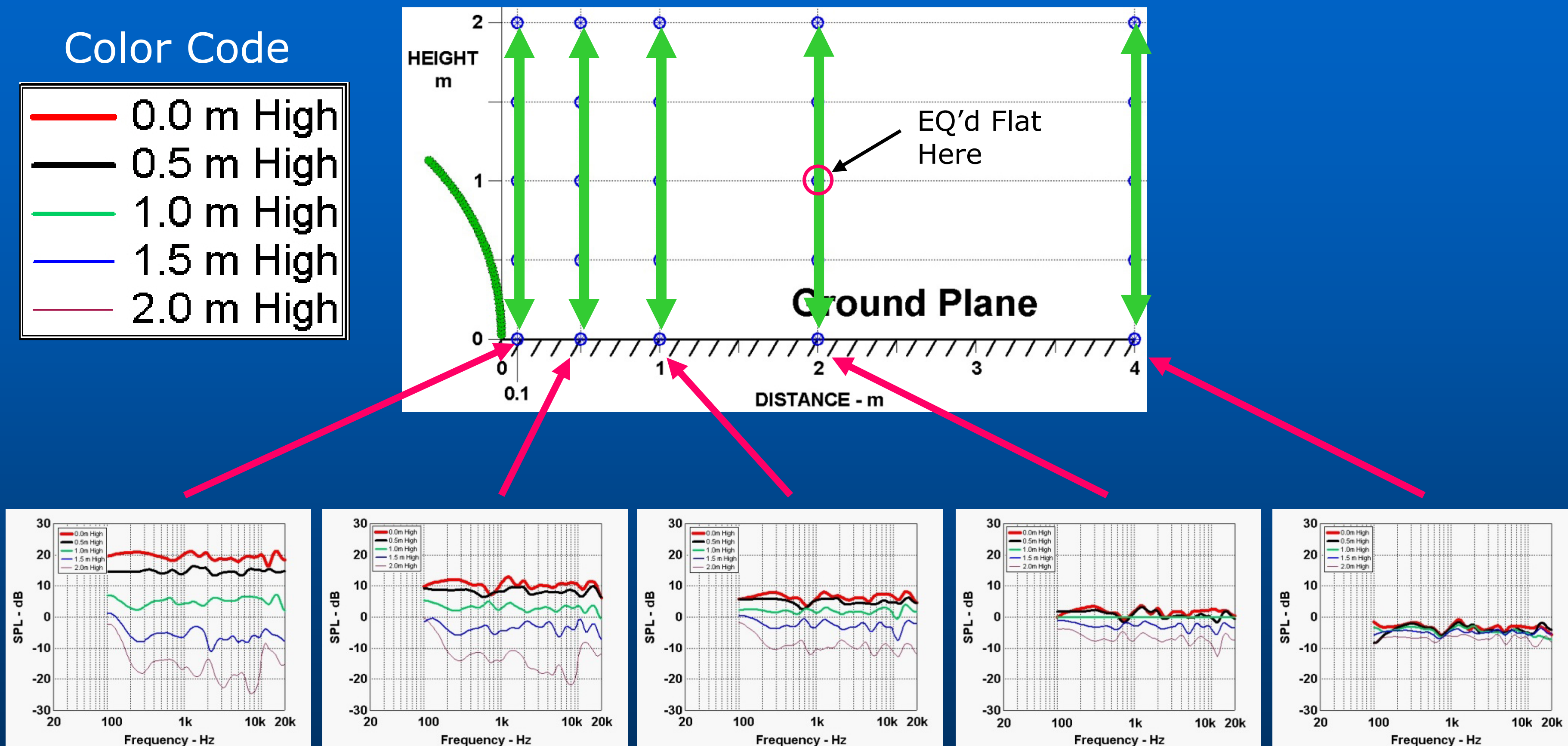

(Normalized to 0 dB at 1 kHz)

# Response vs. Height

## (at Different Distances)

Color Code

- 0.0 m High
- 0.5 m High
- 1.0 m High
- 1.5 m High
- 2.0 m High

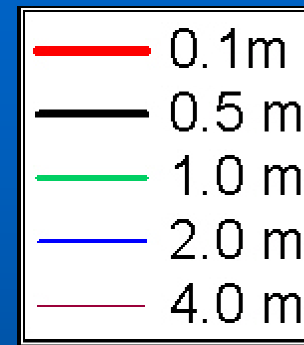



0.1 m | 0.5 m | 1 m | 2 m | 4 m

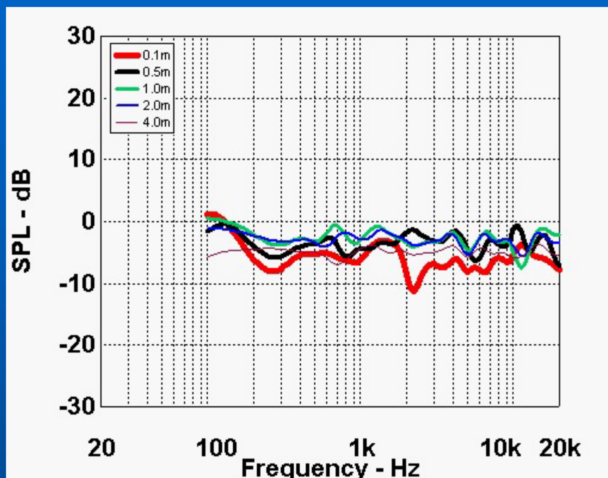
# Response vs. Distance

## (at Two Different Heights)

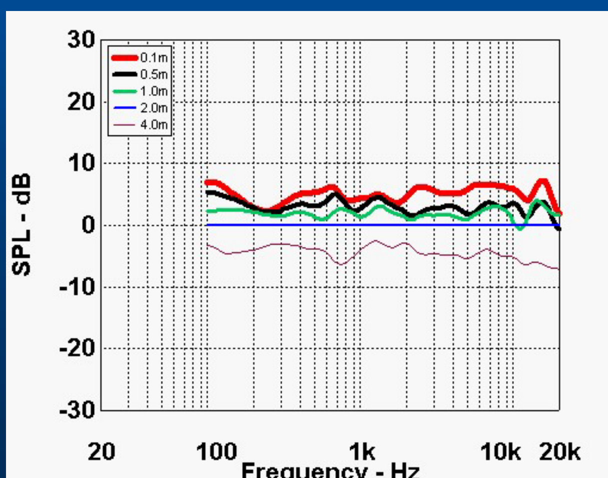
Color Code

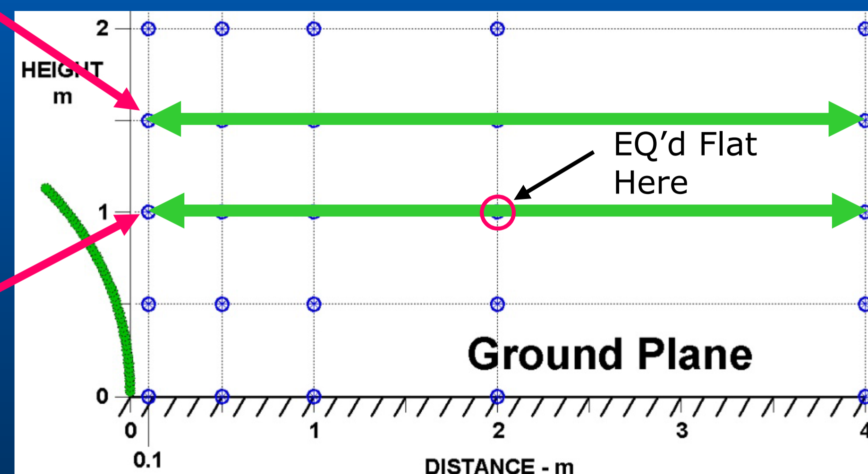



1.5 m High




1 m High

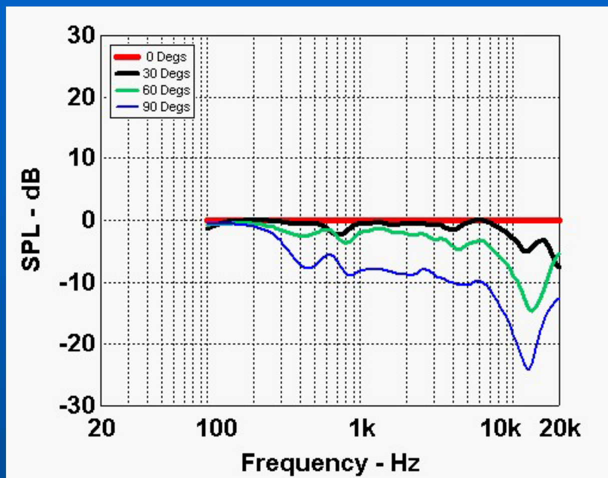


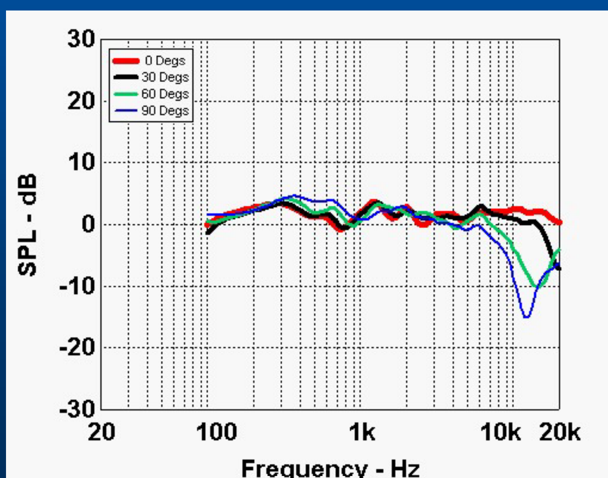
# Response vs. Angle

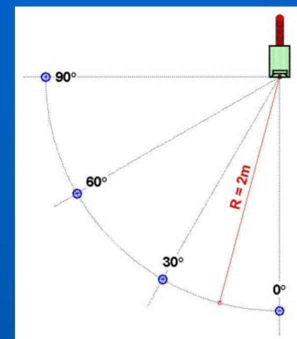
## (at Two Different Heights)

1 m High

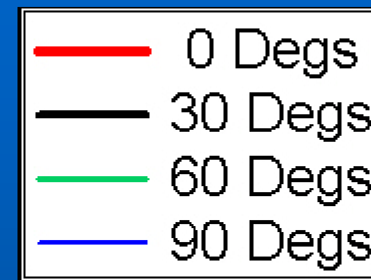


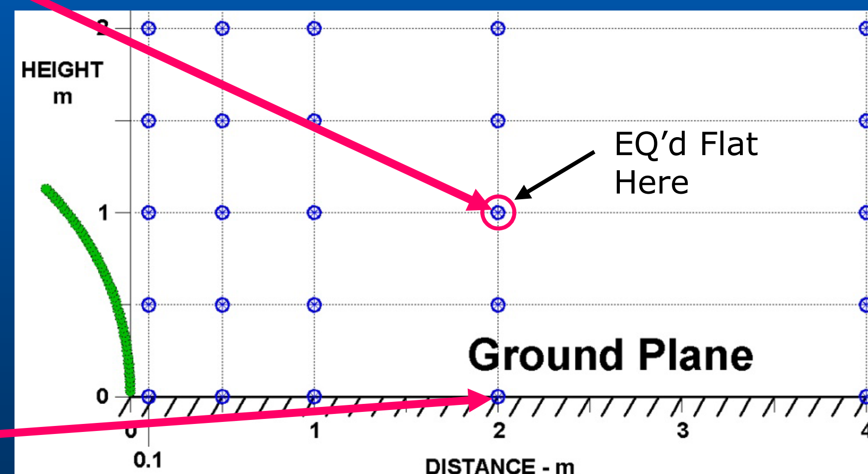

0 m High



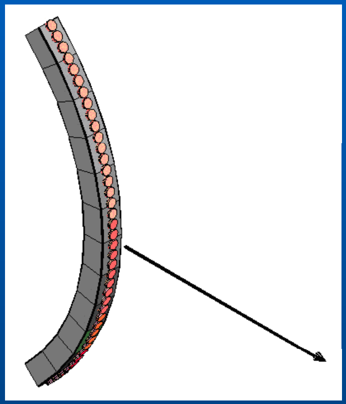

Color Code



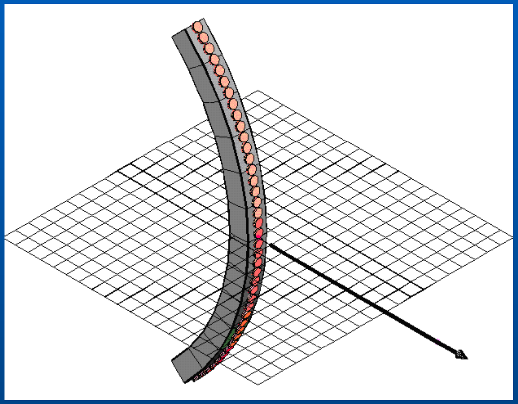



# Working Off the Side of the Pattern

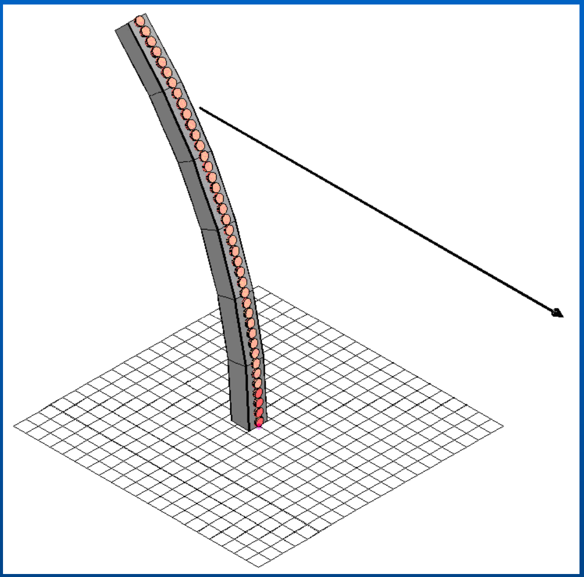
- At specific heights, the variation of near-far SPL is minimized because the trajectories approximately coincide with specific constant-pressure contours.



Main axis of free-standing CBT array.



Main axis of ground-plane CBT array. Axis grazes the ground-plane!



Typical listening axis for a ground-plane CBT array. The listener listens on an axis that is offset from the main axis of the array.

# Constant-Pressure 3-dB Contours

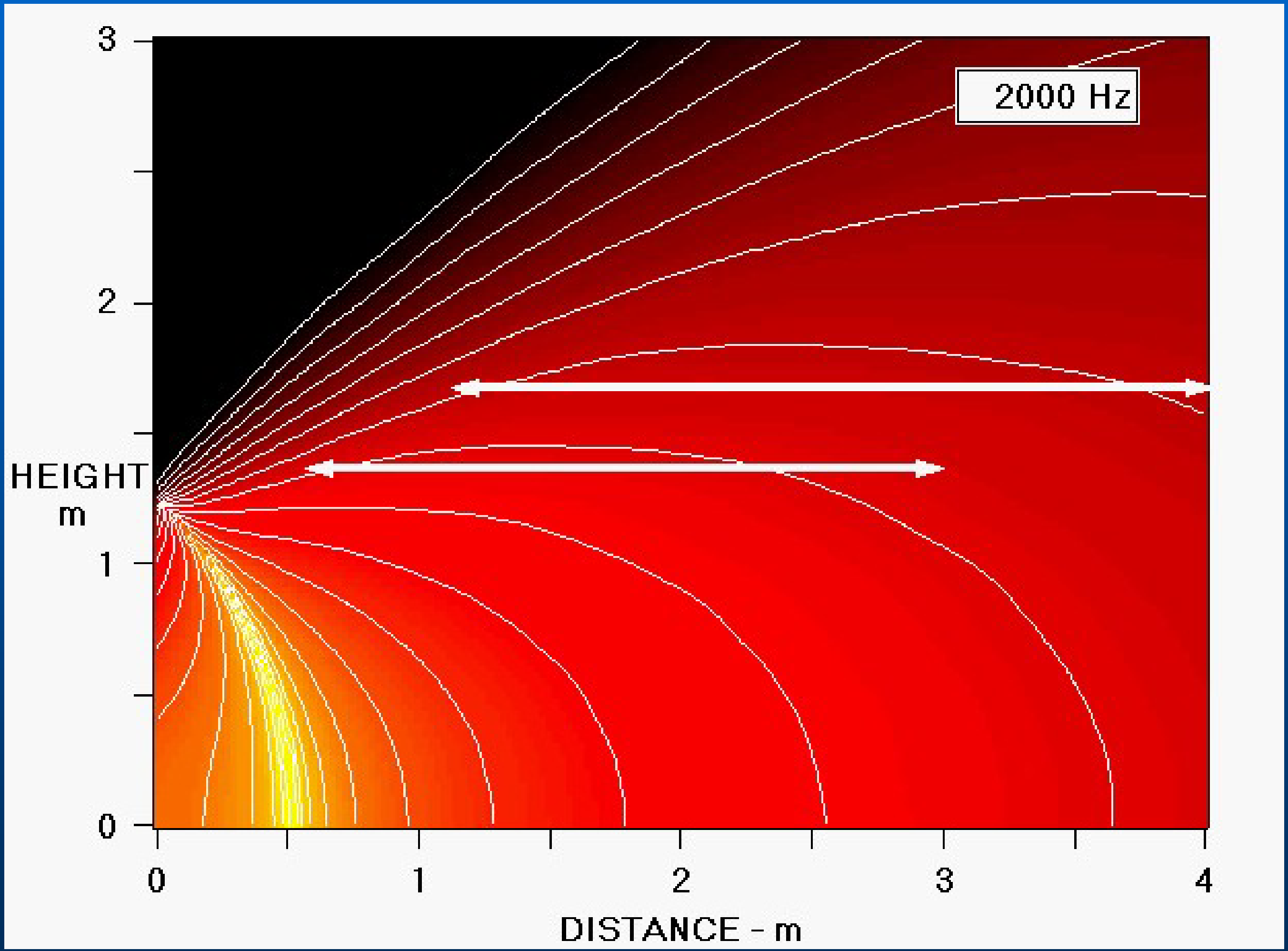
(Horizontal lines are drawn where the SPL is constant within ±1 dB)

Range: 1.2 to 4 m

Height: 1.7 m →

Height: 1.4 m →

Range: 0.6 to 3.3 m




This works quite well because the polars of the CBT array are so consistent with frequency!

# Conclusions: Advantages of CBT Ground-Plane Array

- Minimizes/eliminates detrimental floor reflections.
- Extremely uniform coverage: up-down, right-left, and near-far.
- Can be implemented without DSP, passive speaker-level shading can be used.
- Minimizes near-far variation of SPL at certain heights.
- The beneficial effects of the ground-plane can be taken advantage of in two ways:
  - Increase Effective Size:
    - Doubles effective array height.
    - Doubles array sensitivity (+6 dB).
    - Doubles array maximum sound pressure level (SPL) capability (+6 dB).
    - Extends operating bandwidth down by an octave (or two depending on how the beamwidth is defined).
  - Decrease Physical Size:
    - Can half the physical height of the array but maintain the same performance as the full-size free-standing array when a ground plane is available.
  - Or a combination of the two!

# Thank You!